The Einstein unified field theory completed

A direct challenge to the basic assumptions, theories and direction of modern and post-modern physics

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Abstract: The golden ring to which most physicists aspire is a unified field theory that incorporates all of modern and classical physics. Some scientists and others call this a TOE or 'theory of everything', but it is no more than false hubris to believe that humans could possibly know and explain everything about the universe at this time. Einstein chased this goal for the last three decades of his life, basing his theoretical research on his general theory of relativity. Meanwhile, the vast majority of scientists supporting the other major accomplishment of the Second Scientific Revolution were investing all of their time and efforts to advancing the quantum theory and their quest has been extremely successful. They originally had no interest in a unified field theory. After Einstein died in 1955, his efforts were all but abandoned because of his philosophical stance against the prevalent Copenhagen Interpretation of quantum theory even though he had been one of quantum theory's founders. During the 1970s the tables started to turn and quantum theorists became interested in unifying physics, although not from the foundational principles of relativity theory. They claimed that quantum theory was more fundamental than relativity so they began the same quest from a totally different direction despite their claims to be continuing Einstein's quest. Throughout the development of the ensuing standard quantum model, superstring theory and many other theoretical schemes, quantum theorists have remained resolute in their conviction that the quantum and relativity are mutually incompatible so the quantum must completely do away with and replace relativity once and for all. However, the quantum theory and relativity are not actually incompatible and, in fact, say the some of the same things about the nature of physical reality. When the similarities are fully defined and studied and the basic assumptions behind each of the theories are altered to reflect the similarities instead of the incompatibilities, only then can the point of their compatibility be determined and act as a unifying principle resulting in a completed unified field theory of the type that Einstein once sought. The development of this physical model of reality is not without irony. Not only is the quantum theory incomplete as Einstein argued in EPR, but Einstein's general relativity is also seriously incomplete and true unification cannot be rendered complete at any level of reality until all the theoretical models being unified are themselves complete.

1. Universality of the problem

Introduction

In recent years, there have been many attempts to fulfill Einstein's dream of unification, although those, invariably, would have been rejected by Einstein because they were not based on general relativity as his earlier attempts were. Recent attempts are based on the belief that the quantum is prior to and more fundamental than relativity. Even string theories, superstring theories and branes as far as they go, would have been rejected by Einstein since they are based on the Kaluza-Klein model of space-time. The five-dimensional model of space-time proposed by Kaluza was an extended version of general relativity, while Oscar Klein's modification of Kaluza's five-dimensional relativity model merely tied the quantum to the cylindrical condition set by Kaluza on the fifth dimension. Yet the modern theories based on Kaluza-Klein seem to have dropped the concept of space-time curvature completely from their model at some point during their own development, while keeping the cylindrical condition and the higher-dimensional embedding concept intact.

Einstein and Bergmann adopted the basic Kaluza model without Klein's modification and proved mathematically that any extension (extrinsic) of curvature into the fifth embedding dimension would be of macroscopic extent instead of microscopic. Einstein always believed that the quantum would emerge naturally from his various mathematical models as an over-restriction in the mathematics of his unified field theory. In 1923, after attempting his first unified (he termed it 'unitary' at that time) field theory he wrote and published another essay "Does Field Theory Offer Possibilities for Solving the Quantum Problem." This essay more or less described how he expected the quantum to emerge within with his own continuous field theory during all of his attempts to unify electromagnetism and gravity for the rest of his life. Einstein also wrote to his friend Besso early in 1924 that "The idea that I am battling with concerns the understanding of the quantum-theoretical facts: over-determination of the laws by having more differential equations than there are field variables." (Mehra, 572) So the quantum was part of the overall idea of unification in Einstein's view although he was not seen by others as actively pursuing an alternative view or interpretation of the quantum.

In other words, Einstein tried to solve the quantum problem by a purely mathematical manipulation, which was wrong, and except for a few alterations to this notion, he maintained this method throughput for the rest of his career. He also tried to explain elementary particles on the physical basis of Einstein-Rosen bridges because he believed in a true continuum with no gaps such as found in particles and elsewhere (black holes) which are known by the general name of singularities, but this effort also failed. In other words, he thought that points in space where curvature (matter density) became infinite were impossible and would not appear in Einstein's final unified field theory.

Just as Einstein believed so strongly in continuity, quantum theorists thoroughly and openly reject the very concept of continuity as well as Einstein's concept of the unified or single field as a continuum. Instead they seek his notion of unification through complex systems of point particles, particle exchanges and mixed boson fields, but also talk about the quantum vacuum as if it were some form of background universal field. These views introduce a fundamental paradox into the commonly held quantum interpretation of physical reality since all of the various parts of the overall quantum theory reject the field concept of continuity. After four decades, these theories, known collectively as the standard particle model of the quantum, are not without serious problems and have come no closer to a true unification than Einstein's attempts decades earlier. Yet it seems that everybody has been beating around the bush rather than focusing directly on these questions. Although quantum theorists reject continuity as expressed by Einstein, they all fully accept continuous boson fields and/or a continuous quantum vacuum field populated by an infinite number of virtual point-particles of various types. How is this hypocrisy even possible and how can it be philosophically justified?

The answer is clearly evident. While philosophers and physicists argue over interpretations of the quantum and claim the mutual incompatibility of the quantum theory and relativity, they have either missed or ignored the simple truth that relativity and the quantum say the same thing about the space-time continuum, but they approach the same problem in different ways. In other words, they represent two sides of a real physical duality in nature. Even classical theories of physics and mathematics suffer from the same duality problems although earlier theoreticians dealt with the problems in a completely different manner. Physical space is dual in that it can be interpreted or described as a space of all possible extensions generating a three-dimensional metric or it can be equally interpreted or described as an infinite collection of individual points. Each point is unique and discrete, but the points still constitute a continuum as does the extension or metric space.

This duality completes the physical nature of the space-time continuum in which our material world exists, but this same duality appears again and again throughout science and mathematics under different guises. It affects motion (change in position) in space-time as well as the physical forces that cause motion (or change in position), all forms of geometry and the number line in mathematics.

Mathematical Dualities

mathematics	arithmetic	conceptual	Tensors	Riemann diff geo	Dynamics
infinite	Number line	continuity	symmetric	line-element→0	avg speed = $\frac{\Delta x}{\Delta t}$
infinitesimal	(assumed) no gap	discontinuity	non-symmetric	(assumed) point-element	inst sp = $\lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right)$

Physical Dualities

space	time	EM forces	Newton	Gravity	physical reality	Gen. Rel.	Quantum
extensions	duration	electricity	relative	F = mg	CM or CG	metric	field or vacuum
points	moment	magnetism	absolute		C of rotation		(assumed) point particle

Those places where the duality is incomplete are far more interesting than those places where the duality complete because they show holes in our knowledge of reality, which render the theories that we use to describe reality, including the quantum and relativity, incomplete.

So both paradigms of present science, quantum and relativity, are correct at least as far as they go, but both are incomplete while scientists interpretations of physical reality are wrong because they assume completeness. That is why both seem to be very nearly correct according to the accuracy of their verified predictions while they also seem to be mutually incompatible. The central problem for physical theories of reality is explicit in the simple fact that not all physical situations take account of the duality of space-time, especially in the case of the quantum. In fact, quantum theory misrepresents the geometrical point/extension duality as a discrete/continuity paradox that tends to mislead scientists and research. So scientists have note even considered the possibility of physical reality at a fundamental enough level to overcome the true duality that rules physical reality. This truth means that nature is neither deterministic (relativity) or indeterministic (the quantum) and the quantum theory is every bit as classical as relativity theory.

The Second Scientific Revolution was not about the rise of the quantum and relativity due to two specific experimental results labeled as 'crises'. Those 'crises' were merely the 'façade' or outward face of deeper fundamental problems with the concepts of space and time and their relativism. In fact, the quantum claim that Newtonian physics was overthrown in the Second Scientific Revolution is no more than a propagandist myth that has been used to justify the most extreme views and interpretations of the quantum theory believed by the majority of scientists. Schrödinger even pointed out how silly the quantum mechanical anti-relativistic attitude was by introducing the concept of entanglement since quantum entanglement is neither more nor less than relativity of a sort, although it does not rise to the level of the formalistic relativity of Einstein. These are just more examples of the similarities between relativity and the quantum that scientists have overlooked, yet there is a message buried in these similarities that can be found and used to unify physics instead of just claiming that the quantum and relativity are mutually incompatible and so fundamentally different (as in debates over the false notions of determinism versus indeterminism and continuity versus discrete) that unification is impossible and using that misunderstanding of nature as an excuse not to unify them directly as they presently are, two equally effective, accurate and successful theories.

Looking beyond EPR

The classical nature of the Heisenberg Uncertainty Principle (HUP) is quite easy to demonstrate. While the HUP is supposedly non-geometrical it is actually anything but non-geometrical in nature. Just because it supposedly deals with a single event unconnected (in either time or space) with any other event or action and does not refer directly to any geometrical structure, that event 'collapses' from all possibilities by either entanglement or conscious action, both of which are geometric (extended) in space and time. According to the HUP,

$$\Delta x\,\Delta p \geq \frac{\hbar}{2} \quad \text{and} \quad \Delta E\,\Delta t \geq \frac{\hbar}{2}.$$

All four variables deal with change in the physical world of one type or another. These quantities are normally described as uncertainties (hence the use of the Δ symbol), yet if there were no change or variation either or both of the quantities there could be no reality or probability (whether certain or uncertain) of a measurement, observation or physical interaction which generates the probability, *i.e.* they would both be "nothing" or would not be related in some physical manner that renders them amenable to measurement or observation as a discrete physical interaction. So it is easier and more accurate to interpret the Δ symbol as representing "change in" rather than "uncertainty in" the specified quantity.

In the first equation, an experimenter observes or measures the relative changes in position (representing space) and momentum in the corresponding direction of space (there is an underlying assumption in this that both occur along the same direction in space while denying the possibility that the interaction is geometric in any way) without any reference to the corresponding change in time, as if time were not even passing during the event. So the uncertainty or 'change' in, whichever the case may be, of momentum acts as a form of 'pseudo-time' relative to the 'changing' position, thus mimicking mathematically a completely real physical situation or quantum experiment. In the second equation, where the 'change' in energy represents a 'pseudo-position' in space, the experimenter is either observing or measuring possible changes in time even though the very nature of physical reality requires that changes in time during events cannot occur without corresponding changes in space (location) and vice-versa, so space and time become the 'hidden variables' or better yet the 'suppressed variables' in experiments designed specifically to measure one 'without regard to' and even 'independent of' the other as mathematically suggested by the HUP relationships.

In mathematics, uncertainties always range for 0 to 1.0 while the corresponding certainties range from 1.0 to 0. If the quantities in the so-call uncertainty principles are true physical uncertainties, then $\Delta x \rightarrow 1$ when $\Delta p \rightarrow 0$ and vice versa. These should yield

$$\lim_{\Delta p \to 0} \Delta x \geq \frac{\hbar}{2}$$

$$\lim_{\Delta x \to 0} \Delta p \geq \frac{\hbar}{2}.$$

In these cases the calculated uncertainties would already be normalized and no infinites that need to be renormalized would ever appear. These represent the true limits of the HUP because the accepted limits of zero and infinity are purely mathematical constructs rather than physical constructs. Physical space-time cannot become 'nothing' at some point relative to other points because it still exists at least approximately to within a quantum measurement of reality. The abstract or unnatural insertion of strictly mathematical interpretations into purely physical situations renders nature indeterministic although, in truth, it is not. Indeterminism is thus not a fundamental characteristic of either nature or physical reality as claimed by quantum theorists and philosophers. The mathematical quantities of Δx, Δp, Δt and ΔE are most certainly related (proportional) to true physical uncertainties, but they are by no means the uncertainties themselves as depicted since the 1920s.

The HUP and quantum theory also separate space and time in just the same manner as classical (Newtonian) physics, as opposed to special and general relativity where space and time are combined in a single non-classical space-time continuum. This type of separation of experimental variables to determine which variables in any given experiment are to be held constant or in abeyance while other variables or groups of variables are measured represents an experimenter's choice rather than a real natural relationship. The relationship of the HUP to classical or Newtonian notions of a separate space and time can be easily demonstrated to prove this assertion.

When the two HUP equations are brought together to form a single equation, such that

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \leq \Delta E \, \Delta t \quad ,$$

Planck's constant is subdued or suppressed, such that

$$\Delta x \, \Delta p = \Delta E \, \Delta t \, .$$

Since equating the Heisenberg equations, thus reuniting space and time into a single framework, suppresses Planck's constant such that space (location) and time (location) can be measured simultaneously, the only possible conclusion is that Planck's constant (or a factor of it) is related to how space and time are bound together to form a space-time continuum. Therefore, Planck's constant can only be interpreted as the connectivity or binding constant for space-time under these circumstances. Consequently, the HUP can only be interpreted as an experimental measure of the extent to which changes in spatial or temporal location, either position or duration respectively, can be measured independent of each other at any scale of nature, although the effect is seemingly invariant over larger scales and thus far more prevalent as a contributing factor in experiments at the sub-microscopic scale.

Moreover, the relationship $\Delta x\Delta p=\Delta E\Delta t$ can be used to algebraically derive the characteristic equations of special relativity – time dilation, Lorentz-Fitzgerald contraction, mass increase and energy-mass equivalence – when a factor of $\Delta x/\Delta t\leq c$ (which can be called the Einstein certainty principle) is introduced to further restrict the physical system under consideration. (Beichler,) Which particular special relativity equation emerges depends only on how the quantities substituted into the HUP equations are defined or interpreted. Since there are no specific criteria for determining the values substituted in any application of the HUP, the values used to determine the special relativity and other equations are perfectly valid. Newton's second law of motion, **F**=m**a**, can also be derived in the more general solution where no such limit to the ratio of $\Delta x\Delta t$ is introduced.

Again, we start from the hybrid relationship

$$\Delta x\Delta p = \Delta E\Delta t \quad . \qquad 6$$

When no reference is made to the speed of light, which further delimits or generalizes the physics involved, Newton's second law of motion can be derived. According to the energy-work theorem, the quantity ΔE should be equivalent to a force applied through some 'uncertain' distance, yielding

$$\Delta x\Delta p = F\Delta x\Delta t \quad . \qquad 7$$

and

$$F = \frac{\Delta p}{\Delta t} \quad , \qquad 8$$

which reduces to

$$F = \frac{dp}{dt} \qquad 9$$

in the limit where the uncertainties approach the natural limits of physical reality near zero according to the basic theorems of calculus.

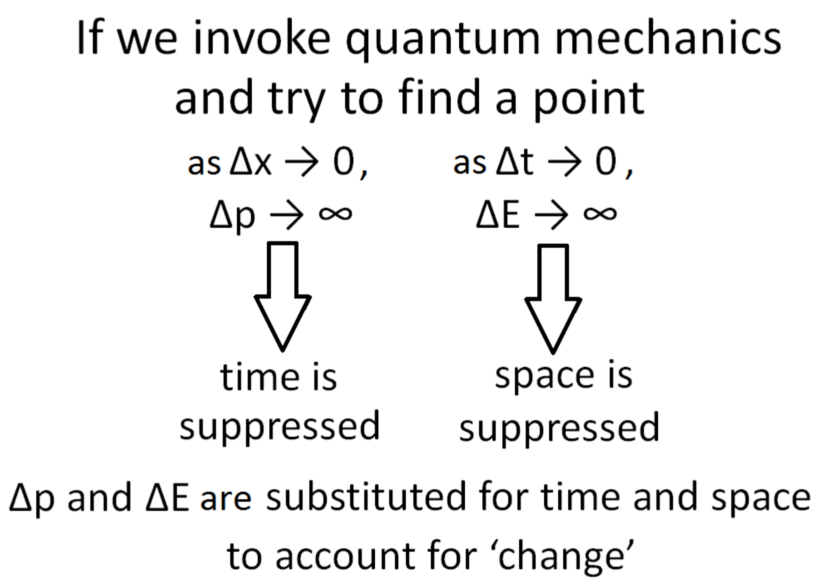
This last equation is none other than the second law of motion as originally stated by Newton. It seems logical that suppressing the speed of light as a necessary factor should yield something Newtonian rather than something Einsteinian. Furthermore, this derivation reinforces the basic idea that the Heisenberg uncertainty relationships do not individually give a complete picture of physical reality. Each of the basic uncertainty relationships that form the fundamental principles of quantum mechanics only gives a partial and thus

> incomplete picture of physical reality. It is only when they are brought together that physical reality emerges from Heisenberg's mathematical statements. Space and time cannot be treated separately as independent variables when addressing physical reality, especially when the changes that occur over the forward movement of time approach their smallest physical measurements in the 'quantum' realm of the microscopic world. (Beichler, 2012, "The Tie that Binds")

These simple algebraic derivations further confirm the interpretation that Planck's constant is the binding constant for combining space and time to create space-time and the constant is subdued or suppressed when space-time is applied in any given classical situation.

The quantum context of space-time

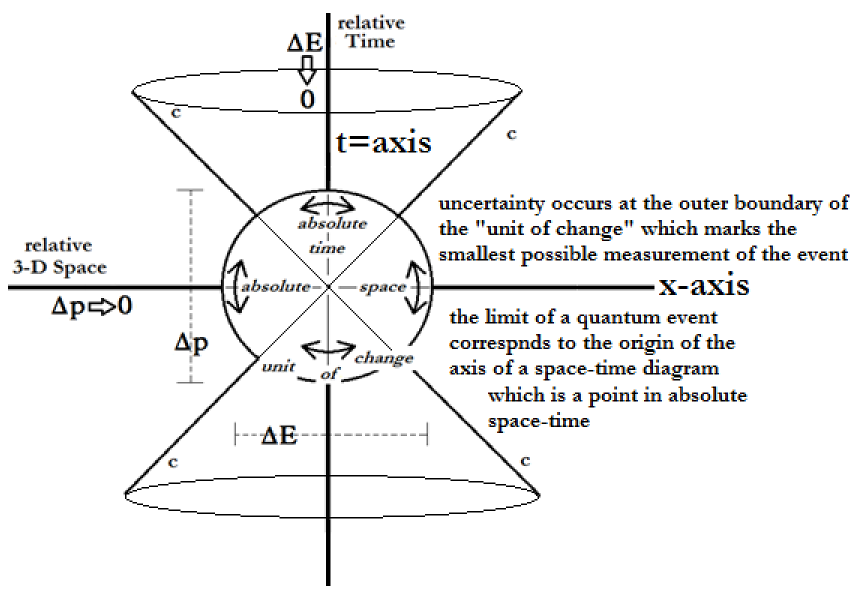
Yet there still remains the algebraic problem associated with any of the variables Δx, Δp, ΔE and Δt when they are allowed to approach zero in the original HUP equations. What exactly does that mean? It normally means that as $\Delta x \rightarrow 0$, when the uncertainty in position is zero and the position is accurately measured, the particle itself is limited to a point in space.



Be this as it may, time and space cannot be suppressed in nature or reality so attempting to do so experimentally in correspondence to the Heisenberg Uncertainthy Principle produces unnatural phsyical situations. Allowing or even forcing the uncertainty, or whatever the quantity actually represents, to go to zero is a thoeretical impossibility and does not imply in any way that particles can be treated as geometrical points. Rendering or hypothesizing that real material particles are geometric points is an assumption at best, and then not a very good assumption, although it is more likely the assumption repersents a mathematical approximation of the physical truth of the matter.

The point-particle explanation does not make logical sense because all real particles are extended and an extended body cannot perfectly coincide with and thus be limited by an infinitesimal point in space. This is an example of how the mathematical concept of uncertainty is treated in the standard model of physical particles, which cannot account for the fact that real particles are extended in three-dimensional space. Real phsyical and material particles cannot be mathematical infinitesimal points, which have no value or characterisitics whatsoever, so the standard model has been based on the illogical and incorrect picture of reality that posits or imagines real particles are points. Since the standard particle model is based upon teh physical consept of point-particles it can never be anything more than a very accurate mathemtical approxiation method used for gaining physical data on particle systems rather than a true explanatory 'theory' of physical reality.

The quantity $\Delta x \rightarrow 0$ actually means that the measuring device being used (a high-energy collision experiment) closes in on the outer material limits of the extended object being measured, rather like a vice clamping down tight on a hard round ball (*i.e.* a proton), rather than implying that the particle itself (the target) occupies or is reduced to a point. If the object beng measured could be reduced to a point, then that point would represent the center of a space-time axis or diagram and no more, but in reality the particle is extended aorund the point.



But the particle cannot be physically reduced to a point, so the measurement taken as the vice closes on the particle would be the smallest measurable 'unit of change' associated with any given experiment. True uncertainty would go to zero when that outer extended boundary of the particle or the 'unit of change' is reached. The radius of that boundary would be proportional to $h/4\pi$ since we would have a situation where the surface area of the sphere around the origin point of the axes is $4\pi r^2$. This would yield the relationship $4\pi r^2 = kh$ where k is an as yet unspecified constant of proportionality that would differ for each and every experimental situation. Of course, the quantity of $h/4\pi$ is already, if not coincidentally, expressed in the uncertainty equations. At that point, the outer boundary of the particle or object being measured would be equal to the smallest possible measure by which the particle or object size could be determined real rather than a true quantum which would appear as a dimensionless point at the axis of a space-time diagram representing the quantum or measuring event.

On the other hand, as $\Delta x \rightarrow 0$ in this case the corresponding uncertainy $\Delta p \rightarrow$infinity by mathematical necessity alone rather than any possible physical reality.

$$\Delta p \geq \frac{\hbar/2}{0}$$

This quantity corresponds to the quantum action along the x-axis while the corresponding case where $\Delta E \rightarrow h/4\pi/0$ to the quantum of action along the time axis of the space-time diagram. Yet there still remains a fundamental assumption in the mathematical operation of dividing both sides of the equation by the value ($\Delta p =$) 0 that $\Delta p \rightarrow 0$ actually has a physical equivalent, but that is not necessarily so except for the stated equivalence of quantum action along both of the axes. Quite literally, there are no real physical circumstances under which the uncertainty in momentum could become infinite, such that doing so is a mathematical illusion that points out the difference between mathematics and physics. The infinite uncertainty in momentum could only occur in a universe in which mach's principle was impossible and, in fact, the interaction under which the HUP holds itself formed its own completely independent universe around itself, *i.e.* the interaction would necessarily become the whole universe.

Any possible physical equivalent to the mathematical operation of dividing by 0 could only exist in a static non-dynamic and non-changing universe, which would collapse if it ever came into being since the mass would always have a non-zero value and the particle would always be moving relative to at least one other such particle (the observer or measuring device) in the universe. A universe in which the physical situation that $\Delta p \rightarrow 0$ would be possible would be a universe that allows no fundamental change at all, which contradicts the stated assumption that Δp could $\rightarrow 0$, creating a sort of Zeno's paradox. So a physical assumption of the possibility that Δp can $\rightarrow 0$, as is ordinary in quantum mechanics, necessitates the use of a mathematical 'fudge factor' called renormalization to fix the discrepancy between physical and mathematical realities. Therefore, the Heisenberg Uncertainty Principle suffers from the same infinitesimal (singularity), zero or point problems as relativity theory and classical physics. It refers only to the zero point, an absolute quantity or position in a Newtonian form of absolute space-time, or the idealized origin of the space-time axes rather than a real extended material particle or body. Given the simple straightforward fact that all areas of theoretical physics suffer from some form of this same problem, the real question that needs to be answered in physics before unification is even possible is 'what exactly is the physical meaning and relationship between the infinitesimal (as a real physical quantity goes to zero) and the actual infinitesimal (geometrical point) itself?'

In reality, uncertainties must conform to another simple mathematical relationship: The certainty and uncertainty of a particular quantity occurring or not occurring must add to one. In other words, the position is either measured or it is not, those are the only two possibilities. If we then define an inverted delta symbol to represent certainty, we have

$$\Delta x + \nabla x = 1$$

and thus

$$\Delta x + \Delta p = 1 \text{ since } \nabla x = \Delta p$$

The certainty in position 'x' must equal the uncertainty in momentum 'p', the quantity with which it is physically coupled or bound in the HUP, and thus uncertainties in position 'x' and momentum 'p' must add to one (or 100%). The alternative would be to admit that the certainty in position is not equal to the uncertainty in momentum, but then there would be no physical basis to couple them together in an uncertainty relationship. Consequently, there is no rule in mathematics or elsewhere that the product of the uncertainties of non-commuting variables (such as Δx and Δt or ΔE and Δt) are limitless (approach infinitesimal and infinite and thus indefinable limits) and need not add to one. Obviously, the HUP does not conform to this mathematical requirement of uncertainties. Once again the mathematical reality represented by the HUP does not correspond to the physical reality, which raises the question of whether the uncertainty principle is about uncertainty at all and in so far as it is about uncertainty physical reality according to the HUP is mistakenly interpreted and renders nature unrealistically indeterministic.

In reality, both determinism and indeterminism are unrealistic in the long run and represent the wrong way to even speak of let alone interpret and understand nature. Whether nature is deterministic as implied by the scientific practice of predicting the outcome of individual experiments from physical theories or indeterministic as implied by the physically faulty mathematics of the HUP is not even a relevant physical question. The only relevance of the question is religious rather than physical and represents a horrid philosophical betrayal of true science. This fact strongly implies the very important fundamental question 'what exactly does the HUP mean?' as well as the question 'do the quantities in the HUP really represent uncertainties?' Even accepting as factual the interpretation that the HUP is actually dealing with uncertainties in some roundabout indirect manner, other factors must be taken into consideration in their physical interpretation.

The only real physical phenomenon where Δx→0 and Δp correspondingly →infinity is special relativity, at least if the Δ symbol is interpreted to mean "change in" instead of "uncertainty in". This change would still represent phsycial limits in that the speed of light 'c" can only be approached and never attained. Picture a *gedankan* experiment in which the width of a proton can be measured with an imaginary mental radar that does not affect the proton as its speed approaches c. The proton width would go to 0 at speed c due to Lorentz-Fitzgerald contraction, but the actual momentum (rather than the uncertainy in measurement, although the difficulty in attempting to measure an infinite momentum would be extremely high) would go to infinity.

$$\lim_{v \to c} \Delta x \cdot \lim_{v \to c} \Delta p = \hbar/2$$

which yields

$$\frac{L_0}{\beta} \cdot m_0 c \beta = \frac{h}{4\pi}$$

$$\text{where} \quad \beta = (1 - v^2/c^2)^{-1/2}$$

And thus

$$L_0 = \frac{h}{4\pi m_0 c}$$

For the case of ΔE and Δt

$$\frac{t_0}{\beta} \cdot \frac{1}{2} m_0{}^2 c\beta = \frac{h}{4\pi}$$

which reduces to

$$t_0 = \frac{h}{2\pi m_0{}^2 c}$$

Calculation yields a value for $\mathbf{L_0}$ of 0.105 x 10^{-15} for a proton, which is very near to (approximately one-tenth) the experimentally measured value for the width of a proton. This value may be coincidental, but then it might also have some deeper physical meaning. This value might even be interpreted as minumum relative uncertainty in measuring the true width of the proton, which seems quite logical given the fact that modern experimental measurements vary from 0.8 to 1.0 femtometers. In other words, no matter what ingenious experiments they develop, scientists may never be able to truly determine the width or a proton with any more accuracy than an error of 0.105 femtometers and that error would be due as much to special relativity as it is to the HUP. In other words, this gedankan experiment clearly and decisively demonstrates the probability that there are deeper and more fundamental similarities between the HUP and relativity than previously known or even suspected.

So it seems that the HUP and relativity theory have far more in common than has previously been admitted or even suspected. If nothing else, they both represent strictly one-dimensional measurment problems and approximations to real physical events in three-dimensional space. Simply put, the central problem for all of physics is the difference (as well as the similarities) between the concepts of point and extension and nothing else. Debates between scientists and/or philsophers whether nature is discrete or continuous at the most fundamental levels of physical reality are just 'straw men' erected by scientists who have not yet grasped the real central problem in fundamental physics, the mathematical distinction between point and extension. Furthermore, it is no coinccidence that the same problem is essential to understanding mathematics in both of its primary fields of number (arithmetic and counting things) and geometry (measuring things). The problem of mentally comprehending the concept of zero (whether it represents a classical Greek 'no-thing', a something that has a value of nothing or an infintesimal) and the infinite as opposed to real numbers (amounts) of real things renders mentally based matehtmcis a good but not a perfect system for interpreting nature and phsicla reality. So mathematical interpertations of nature are not necessarilly equivalent to physical reality itself. and should be taken with a grain of salt in some cases.

The corresponding problem in mathematics

While the point problem has never been solved in mathematics, once it is understood in physics solution to the problem should become obvious. The mathematics normally used by physicists and scientists is an idealized mental construct that describes nature or physical reality. In other words, no matter how hard mathematicians try to 'rigorize' mathematics by stripping mathematical systems of all references to physics and the phsyical world, mathematics in the end is still a product of the human mind and the human mind is a product of nature and the physical world. So mathematics cannot be completely 'rigorized' or stripped of its connections to the physical world. That, however, does not mean that every mathematical system and theorem must have an exact physical correlate. Different mathematical systems based upon specific theorems do not necessarily equate to real physics, so thay can only act as guides for phsyics and science. Mathematics could never replace physics as a science or as more accurate than nature itself. Since mathematics has been rigorized, mathemical modls of nature can only offer suggestions on how the the physicist who wants to describe the physical world might actually describe it.

To find the solution to the point problem, teh first instinct is to turn to mathetmics as it spplies to physical reality for suggestions. In the differential geometry of a surface (mathematics) or space-time continuum (physics), the line-element (area or volume) is defined as $ds \to 0$ where $ds = \sqrt{(dx)^2}$. This yields

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2$$

for a three-diemsnional surface in Riemann's differential geometry and

$$\lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

in the dynamics study of motion called teh calculus. this is all that is needed to guarantee internal consistancy of the mathematical systems, otherwise these formulas just list teh reuirements for the existence of continuous surfaces and unbroken lines of motion. so indiviudal points along teh surface or the line of maotion are not uniquely defiend even though they must exist in nature. In other words,

We know that individual points exist in space-time, but how do we even locate an individual point? Given that

$$ds = \sqrt{\sum (dx)^2} \text{ and } \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt} \quad ,$$

and a line consists of an infinite number of points, we cannot even build a simple line from two or more points .

So while we can imagine calculating a limit of a physically extended 'thing' approaching zero in extent but never really reaching zero, the mathematical opposite should be true but either is not or has never been tried before. We should be able to take any number of points and put them together to consruct an extened 'thing'. Yet it seems that this cannot be done or even imgained for the sake of mathematics let alone physically.

Every line and extended suface contains an infinite number of points, no matter how small. So as our objective approaches a single end or limiting point, it still consists of an inifinite number of points itself up until it goes to zero even though we cannot go from an inifinite number to nothing while remaining continuous. However, if we try to build a continuous extended surface of as many dimensions as is necessary out of individual points, such as a line, we cannot do so because the individual infinitesimal points have no dimension and when we put two, three or even an infinite number of them together, they all fit into the one single discontinuous point. So a single discrete point causes a break or discontinuity in the line or surface, while that same individual point can absorb and internally hold an infinite number of other such points without growing or extending outward, which implies that it is continuous with the line with which is forms a discontinuity in the sense that both the line and the point hold an infininte numebr of points in them. In other words, it would take at least an infinite number of points plus one, where infinity plus one is not equal to infinity as is normal in mathematics, to construct a line of two points length from scratch. A true discrete point is singular and thus only a true discrete point can break the continuity of a line or surface, but how can a true discrete point be defined in such a way that it is discrete without being the point of discontinuity, *i.e.* so it can be one of the infinite number of points that constitute a line or surface? This problem needs to be overcome to solve the conceptual discrepancy between point and extension as well as the discrepancy between mathematics and physics necessary to unify physics.

The ancient Greek philosophers carried on a similar and indeed related debate over the differences between nothing and 'no-thing', which is why they never accepted the concept of the number zero. The Greeks did not accept the number zero because a true 'no-thing' (being discrete) would not even have a value of nothing, the value being a property even if it is nothing, and therefore a zero should be a 'no-thing' and not just a nothing. A nothoing was something, even if only for the fact that calling a thing nothing gave it a name and only something can be named and thus not be a 'no-thing'. Today, the value zero (in mathematics) and a geometrical point (in physics) are treated interchangeably, sometimes as 'no-things', as is the case in a quantum point-particle that can have no properties by which to physically distinguish it, and as nothings which are something or things with a value of nothing, but still things and therefore not 'no-things'. Quantum theorists also treat point particles in this second manner when they keep adding particles with no value to their probability calculations using Feynman diagrams. The practice is supposed to do away with renormalizations, *i.e.* unwanted infinites in the calculations thaqt can onlhy be gotten rid of by fudge-factors, but all it really does is substitute the imposition of a system infinitesimal point particles for the fudge-factors of renormalization. The more point particles they add, literally nothings to nothing, the better their calculations of probabilities but the harder the mathematics to make the calculations. They are just piling points into points, without regard for their physical nature, *i.e.* they are using nothings as if they are 'no-things'. So it seems as though the ancient Greek philosophical differences between 'no-things', which are truly discrete and thus have no value associated with them, and nothings, which are discrete but have values attahcd to them, have been forgotten in both mathematics and physics.

So the mathematical and physical situations are clearly different in these cases. As far as application of the mathematics to the physical world is concerned, the latter expression forms the theoretical basis of Newton's calculus of fluxions (although with some philsophical exceptions) as well as the philosophical basis of modern calculus in mathematics. In each physical case, we are looking at limits approaching zero, which differs philosophically from mathematics since mathematics allows the quantities to actually reach zero ($1/\infty$) and infinity, but does not really understand what that means, while nature and thus physics abhors infinitesimals of any kind. In the case of Newton's thoery of fluxions, the limit of $\Delta t \rightarrow 0$ could never be reached because absolute time could never equal zero. Relative time could go to zero, but absolute or universal time would always have a minimum of 'nothing' or 0 that moved time forward. When mathematicians 'rigorized' calculus by their definition of instantaneous speed as the limit as $\Delta t \rightarrow 0$ of the average speed, they literally threw Newton's concept of the absolute forward motion of time out of physics and the legend that the laws of physics did not distinguish between forward or backwaard change in time was born.

So in the end the geometrical, dynamical and quantum problems are all the same, except for their philosophical and physical interpretations. Even when geometry is applied physically, problems ensue in the form of mathematical singularities popping up in physical situations, *i.e.* protons, black holes and the Big Bang, which correspond philosophically to the problem or concept of infinitesimals and infinities in the HUP that can only be fixed by the synthetic practice of applying renormalization. In the standard model of quantum theory where Feynman diagrams are used quite extensively, the Higgs particle is no different. As a true discrete point particle the proton can have no physical properties such as mass. So a new exchange point particle, the Higgs particle, was postulated (if it were physics instead of mathematics it would have been hypothesized instead of postulated) to carry mass to the discrete proton whenever it moves through the universal Higgs field. This notion further complicates the point/extension problem of physics by its failure to differentiate between a 'no-thing', like the truly discrete proton, and a nothing like the Higgs particle, an infinite number of which virtually constitute the Higgs field. If the Higgs particles were truly discrete, they could not constitute the Higgs field, while calling them virtual merely circumvents and obfuscates the conceptual problem. Other applications of the point concept to physics also suffer from this same problem, further complicating the point-extension problem.

Even in the normal practice and application of quantum mechanics, a specific mathematical method for getting rid of zero points is used. It is called perturbation and by its very nature it turns the assumed point location of a quantum event into a probabiistic approximation of that point. In other words, the mathemtical method used introduces the probability and indeterminism into nature rather than interpreting nature as probabilistic or indeterminate. A zeropoint cannot be physically measured because it represents a dimensionless disconinuity, whether a 'no-thing' or a nothing. So the zero point undergoes a mathematical 'perturbation' which converts it to a 'measurably' extended object with dimensions which amounts to is the mathematical equivalent of 'smooshing' a point out to create a continuously extended surface or body. The mathematical process of integration can then be conducted (integration only works over continuous lines and surfaces no matter what the dimensionality) even though the integration was not possible over the original point because it represented a discontinuity.

Needless to say, all of these problems are abhorrent to physicists, but accepted in large part because the quantum is so well misunderstood while these mathematical (rather than real physical fixes) gimmicks lead to unnecessary and incorrect discrepancies between modern and classical physics, so they should be condemned as bad physics and on occasion have been. If the physics of the quantum was better understood, perturbation, renormalizarion and other mathetmical gimmicks that are commonly used to mathematically approximate physical reality would not be necessary. It is in this manner that quantum mechanics is incomplete although inadequate to measure physical reality would be more accurate. However, quantum emchanics works so well because quantum mechancis is an approximation method that can be internally adjusted (thorugh normalization and such) by the mathematical methods employed to approxiamte and thus accurately simulate physical reality.

As bad as this situation is for physics and mathematics, there is a simple solution to the problem that has been implied but never attacked in the work of countless theoreticians and mathematicians for more than a century or two, if not longer. However, those researchers have never correctly or even approximately defined what the central problem (what is a mathematical point and how can it be represented in physical geometry) is in any manner that would lead to a solution of the problem. In other words, both physicists and mathematicians have been 'beating around the bush' and 'circling' the answer to the central problem in each of their disciplines without ever really being completely aware of and stating the central problem. They have been skirting this crucial and fundamental subject – the true nature of an infinitesimal or geometric point – without trying to tackle the problem head on and solve it. Yet a very simple solution can be developed and when it is the fundamental theories of physics can be simply uniifed

II. The relative nature of the problem

Basic unified field theory

To many physicists it would seem that the unification of physics within a single paradigmatic theory has been the primary goal in science for only the past few decades, but this would not be true. Unification was the original goal of Einstein and a few other physicists throughout the 1920s to the 1960s during a period of time when quantum theorists were ironing out their own unique set of problems. Unified field theories based on general relativity were 'almost' but never quite that popular between 1918 and 1960. They were overshadowed by the growth of quantum and nuclear physics which had no need to unify with gravity (and electromagnetism) during the middle era (1927-1970) of quantum development. Relativity based unification is usually represented as attempts to develop a single theory based on a unified field from which both gravity and electromagnetism emerge as equals, but this interpretation of history is a 'phallacy of fysics'. (Beichler, 2014)

Attempts at unification, especially those pursued by Einstein, were attempts to derive an even more general geometry of the world (than Riemannian geometry) that could include both electromagnetism and the quantum. More precisely, Riemannian geometry was based upon metric extensions of space to form surfaces of any number of dimensions and not points, while a more general geometry would reflect the contributions of both points and extension to the surface. Accounting for points in the continuity of the continuum used to express gravity in general relativity automatically opened the door opens the door to including the standard point and other models interpreted as various quantum theories.

On the other hand, the unification of physics under the guise of the quantum paradigm only emerged during the 1970s and has since overshadowed all attempts to unify physics from the fundamental principles of relativity. Far too many modern physicists believe, without any true supporting evidence beyond their own biases and opinions, that the concept of the quantum is far more fundamental than relativity so earlier attempts based on the continuity of relativity have been all but abandoned. The fundamental nature of the quantum theory, that all of nature is discrete in its most fundamental and smallest units, is an assumption that does not hold when describing reality in physical theories. However, both approaches are basically flawed because both relativity and the quantum theory are incomplete as they now stand which proponents of both sides of the debate interpret to mean that relativity and the quantum are mutually incompatible. In this case, only one or the other of the theories, which automatically excludes the other, could be used as a basis for unification of the two theories. This approach to the problem of unification is mere rubbish even though it only reflects the nature of how incomplete the two theories are. Had either side of the quantum versus relativity debate just simplified their worldview and sought commonality between the two, unification would have been accomplished long ago.

The point is, literally, that the discrete quantum, continuous relativity, basic physical geometry and classical physics all share one common characteristic, the paradoxical and thus problematic duality between a dimensionless point (which is conceptually discrete, or so we have been taught) and an extended length (continuity) in any dimension. If the problem of unification is approached from an understanding of how this paradox relates to each paradigm, all of physics could be unified under a single new theoretical paradigm. Unfortunately there has never been a method, either mathematical or physical, by which a three-dimensional space can be generated from two or more dimensionless points. This shortcoming raises the question "how can the dimensionless point-particles of the standard model which presently dominates physics be extended to account for the three-dimensional space in which the physical interactions they describe occur?" Fortunately, this question can now be answered, but the answer does not favor the standard model of the quantum as it is presently interpreted. Instead, a unified field theory based on continuity that completely unifies the quantum and relativity and completely incorporates the standard model as well as the superstring, brane, quantum loops and quantum gravity as well as other suspected quantum models and commonly accepted classical theories has now been completed.

The fundamental nature of continuity

Einstein's development of general relativity came as a huge surprise to the physics community in 1915 because physicists were expecting to explain matter on the basis of electricity (Mie's theory to mention only one popular theory of the time) rather than gravity as Newton had done. Einstein actually corroborated Newton's approach to defining matter by gravitational attraction rather than electricity by equating matter to the local curvature of space-time in a Riemannian surface. So Einstein's new theory of matter, or rather of the implied gravitational curvature structure of material particles, flew directly in the face of the trend to define matter and material particles as small bits or quanta of electricity.

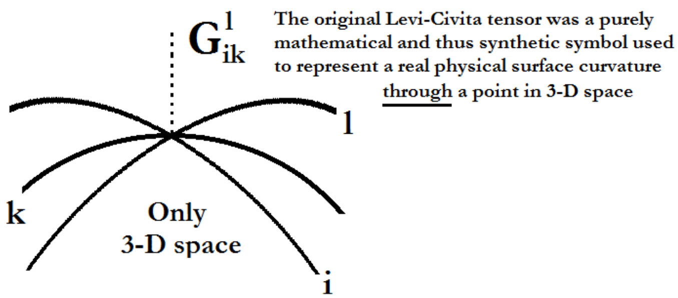
Einstein did not at first believe that his new theory was incomplete in any way, except perhaps to account for the quantum. He thought he had already included the electromagnetic field as part of the stress-energy tensor, so the first work on unification resulted more from the observation of others that the Riemannian geometry used by Einstein in general relativity was itself incomplete. Thus the first unified field theories developed to unify gravity and electromagnetism were really more attempts to render relativity even more general by expanding the geometry than they were attempts to include electromagnetism. The two trends just happened to come together rather quickly and perhaps even unfortunately. Yet the fact that these researchers believed Riemannian geometry was incomplete is the really important factor that needs to be taken seriously while the geometry is further developed to actually unify all of physics. In other words, these researchers did not go far enough in developing a new more generalized geometry than normal Riemannian geometry to determine the real basis for unification.

When Einstein developed general relativity in 1915 he used a Riemannian metric geometry as expressed by Levi-Civita and Ricci-Curbastro's tensor calculus. Although he was still interested in developing the idea of the quantum, being a founder of quantum theory, Einstein thought general relativity to be the greater need in physics and expected to settle the quantum question later using general relativity. Since matter was depicted as a very extreme curvature in the space-time continuum, particles would necessarily have some type of boundaries where the curvature settled down to a nearly or approximately flat configuration that could be equated to the very weak force of gravity spreading outward from the outermost boundaries of particles. Since particles were pictured internally as a form of extreme curvature in the space-time continuum, particles would necessarily have some form of quantum determined boundary that separated them from the surrounding curved or non-material environment of empty space or the rest of the continuum. Modern quantum theorists (especially when using continuous quantum fields) run into this same problem, but have been able to ignore it by defining particles as dimensionless points in space-time surrounded by either ill-defined and/or undefined boson fields that interact with the point particles by the exchange of other point-particles called bosons. These quantum point-particles would correspond roughly to the singularities (zero-points and thus discontinuities) that Einstein's tensor calculus placed at the center point of the space-time curvature representing real material particles.

In Einstein's theory, the curvature of the space-time continuum was instead characterized by placing tensors at each and every point along the curvature. Tensors at any specified point in the space-time continuum noted the change in surface curvature in each of the three dimensions of space through that point. The system of tensors thus created a calculus that could be used for calculational purposes as material bodies or light waves traveling across the curved surface of the world. However, Einstein's physical interpretation of the Riemannian mathematics inspired immediate challenges to the fundamental assumptions of Riemannian geometry itself by other researchers. Each noted in his own words that the tensors only represented changes in the curvature through the infinitesimal geometric points of space. In other words, it was not until Einstein's successful application of Riemannian geometry to gravity that others realized Riemann's generalization of geometry was not complete. Noting that discrepancy, the mathematicians Hessenberg and Levi-Civita were inspired by the successful utilization of Riemannian geometry and tensors to explain gravity by Einstein to expand Riemannian geometry in 1917. The physicist Hermann Weyl also began his unification in 1917 from a strictly mathematical expansion of Riemannian geometry. He only introduced his physical concept of gauge at points within an affinely connected space corresponding to his more generalized geometry to unify electromagnetism and gravitation in 1919.

In other words, the first wave of unification was not so much about a philosophical or even a physical unity between the primary fields of electromagnetism and gravity as it was about generalizing the geometry that Einstein had used to explain gravity more accurately than Newton. The mathematical expansion and generalization of geometry just provided physicists a convenient way to include electromagnetism in the field equations, but they missed the real problem with the new concept of physical space, so all such attempts at unification were fundamentally wrong.

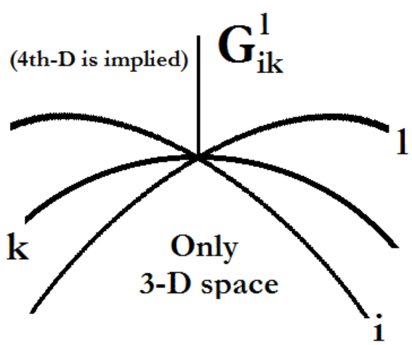
Einstein tensor in 3-D space



They literally missed the point, even though they utilized the geometrical concept of a point as a catalyst to develop the non-Riemannian geometries with which they filled the point. All of these men noticed that the tensors used to represent the metric curvature of space-time was located at a point of space, but only took account of the continuity of the curvature **through the point** rather than any specific physical characteristics of space-time **at the point**, which is only the beginning of the correct approach. Under these conceptually incorrect circumstances, each of these men filled the mathematical point itself with a new geometry that differed from the Riemannian geometry used by Einstein to explain gravity as space-time curvature and thus the non-Riemannian geometries were born.

Weyl immediately sought a physical interpretation of the points themselves and developed his gauge theory in an affinely connected space. His model thus became the first of the unified field theories that sought to unify gravity and electromagnetism within a single field structure. In these structures, the Riemannian metric (based on spatial extensions) was used to explain gravity just as Einstein said, while the new non-Riemannian geometries in the points of space were used to express the electromagnetic field of Maxwell and Faraday.

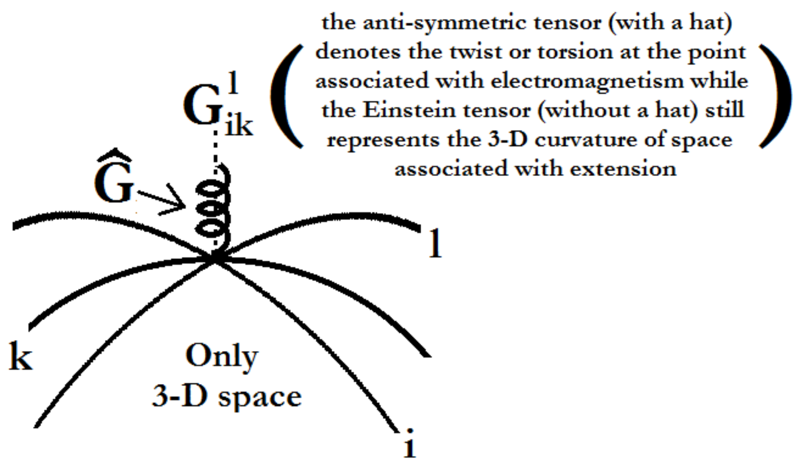
The incomplete nature of the non-Riemannian fix



The Euclidean, non-Euclidean and Riemannian geometries all suffer from the same problem (the empty point), but the non-Riemannian geometries (or tangent spaces) tried to fix this problem by inserting new geometries 'into' or 'at' the point. Unfortunately, they did not simultaneously connect the point to other points along the curvature in 3-D space. So the non-Riemannian geometry in the point is STILL disconnected from the geometry along the whole of the metric that describes the space-time continuum

This point was emphasized by Pauli who classified all such geometries as 'tangent spaces' or alternate space within individual points in the Riemannian curvature that were tangent to the Riemannian curvature at each and every point along the metric.

The astronomer Arthur Eddington is better known for his confirmation of light bending during an eclipse at Tenerife in 1919, but he followed Weyl and adopted Weyl's unification theory based on the mathematical concept of an affine connection. Although the point was next taken up by Eddington in 1921, Einstein did not follow suit until 1923 because he was looking only at the physical consequences of electromagnetism for his general theory of relativity rather than generalizing the mathematics and he thought his curved space-time platform adequately described the motion of charged particles within the concept of the stress-energy tensor that determined the curvature of space-time. Einstein was not yet convinced, at least until 1923, that the Riemannian geometry was not yet generalized enough for a complete picture of gravity and electromagnetism. He then adopted the wrong approach to unification that others had taken by equating the non- or anti-symmetric tensor corresponding to a point in space as the correct manner in which to express electromagnetism, leaving Einstein's metric tensor to describe gravity..

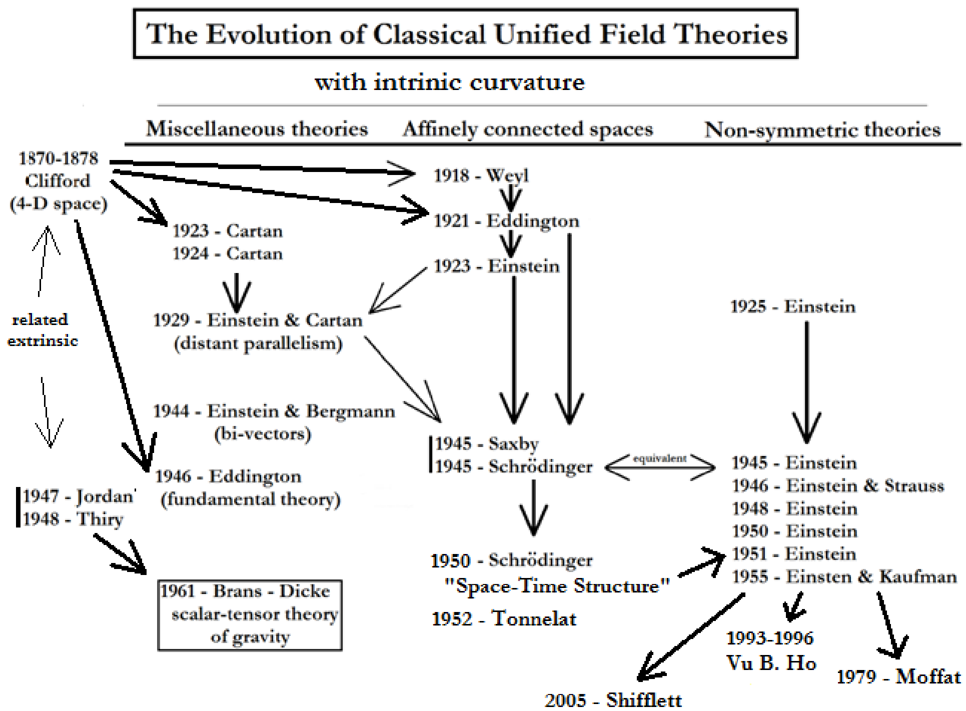


Yet no satisfactory conclusions on how to combine the electromagnetic and gravity fields into a single unified structure or final theories were ever reached. All of these theories sought to unify the known natural forces within an intrinsically curved space-time continuum (whatever that meant to each theoretician) even if a higher dimensional extrinsic curvature was implied by the structure, but they all failed none-the-less. Even the few attempts made along these same lines since Einstein's death have also failed utterly.

When Einstein finally took up the search for unification, he saw it more as bringing both electromagnetism and gravity within a single field structure rather than a way to mathematically develop a more generalized world geometry and thus missed the point. Meanwhile, the mathematician Élie Cartan developed an alternative geometry in 1923 and applied the geometry to physics in 1924 in his own unification attempt. Cartan's new geometry was based upon his concept of torsion which he had developed in 1913. This new geometry introduced the concept of torsion (Clifford's original word for the concept was twist, 1873; 1876) which entered relativity as a new anti-symmetric tensor. Electromagnetism was already considered a torsional field as opposed to gravity which was not, so the anti-symmetric tensor was used to represent electromagnetism. In a sense, the mathematical description of a physical space was dualistic whereby Riemannian geometry and Cartan's geometry represented the two parts of the dualism. Physical space corresponded to a metric (extension) Riemannian geometry characterized by linear displacements through space (which became the shortest geodesic across curved space-time) while it simultaneously corresponded to a torsional (point) geometry, as described by Cartan, which was characterized by parallel displacement. In other words, the individual point by individual point torsion resulted in a torsion field that was anti-symmetric as opposed to the Riemannian metric which was simply symmetric (metric). The anti-symmetric portion of a tensor was thus wrongly thought to represent the electromagnetic field in all of the attempted unification schemes that were developed upon the notion of intrinsic curvature.

All of these attempts correctly noted the duality of space in some manner or fashion, but wrongly utilized that duality in a physical sense by equating the duality to electromagnetism alone. So, all of these attempts were doomed to failure, including Einstein's later attempt to unify gravity using a non-symmetric tensor (1925, 1945 and onward) as well as a short-lived attempt to use bi-vectors (1944) to represent the geometry at the point more fully. These efforts at unification were correct in noting that the geometry was incomplete at individual points in space, but they misinterpreted the fundamental dualism of the point/extension geometry as related to the forces rather than space-time itself. According to this interpretation, gravity and electromagnetism should each have components representing the same duality of point and extension rather than just one or the other.

This whole historical line of theoretical research was fooled by a single point of geometrical origin, and this historical fact has never been studied or analyzed properly. Instead, Einstein and other's attempts have been interpreted as representing unrelated individual models that sought to unify electromagnetism and gravity, but not the quantum (which could have been associated with the points), blindly jumping from one hypothesis to another or from one mathematical gimmick to another, rather than the continuous progression of valid ideas that were used to express and incorporate point-elements into the extension-element geometry of the world.



When seen in this light, Einstein's search for a unified field theory takes on a new light and a whole new meaning for physics. There is simply a simple method to his reported madness. Even though these attempts were all doomed to failure from the very beginning, they actually started from the promising and correct premise that the geometry in the points could differ from the metric geometry of three-dimensional space passing through the points because electromagnetic theory is already split into two tensors, symmetric and non-symmetric (skew- or anti-symmetric). The metric portion of space and thus the symmetric tensor representing the point in space is electrical in nature while the non-symmetric portion is magnetic in nature.

So Einstein and others' attempts to use anti-, skew- or non-symmetric tensors to represent the electromagnetic field failed utterly for one and only one specific reason. Electromagnetism already accounted for its non-or anti-symmetric half (magnetism) in its basic formulations, which put it on a higher level of mathematical and physical sophistication than gravity theory. The duality of electromagnetism was necessary to explain how electromagnetism worked given the duality of space itself. Since space (and time or space-time) is dualistic rather than electromagnetism (or gravity for that matter), space (and time) needs to be simultaneously represented by either and/or both symmetric (extension) and non-symmetric (point) portions by a single tensor. This is the point that everyone missed and all of the theoretical model have faltered on. The symmetric tensor through points in space only represents the metrical properties of space that are shared by the electric and normal (Newtonian and Einsteinian) gravity fields, while the non-symmetrical portion of the tensor represents the point-properties of physical space inherent in the magnetic field and an as yet unknown and unsuspected new gravitational force factor.

The dualistic nature of electromagnetism

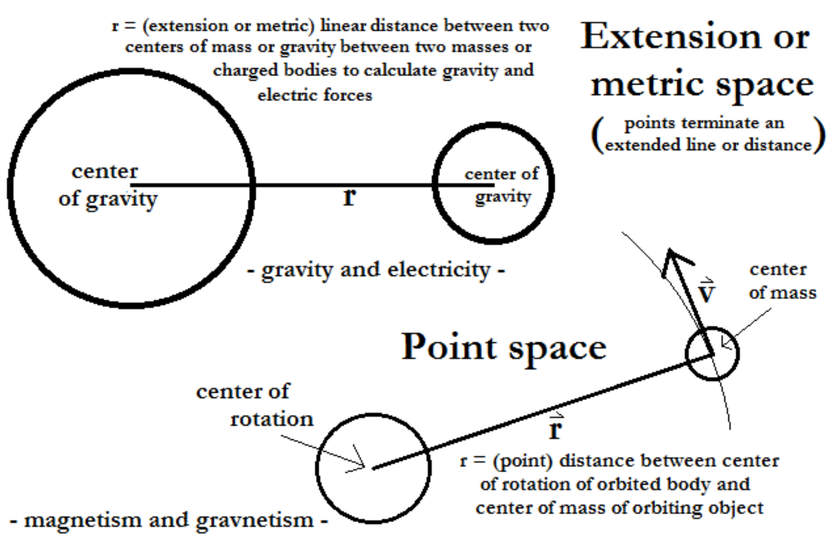
Electromagnetism was already known to have two parts, one of which is symmetrical and the other which is anti-symmetrical or torsional. So electromagnetism already accounts for the dualistic nature of geometric physical space in which it acts, even in its classical form. These two parts are both represented in the classical Lorentz equation of electromagnetic forces.

$$\boldsymbol{F}_{em} = q\boldsymbol{E} + m\boldsymbol{v}\otimes\boldsymbol{B}$$

The first component $q\boldsymbol{E}$ is the force relative to extended-space (metric) or electricity, while the second component $m\boldsymbol{v}\text{x}\boldsymbol{B}$ represents the force corresponding to point-space or magnetism. Electricity and magnetism are not two separate forces, although for simplicity sake they are usually treated as such. They are two faces of a single force which emerge when that single force interacts in two different (symmetric and anti-symmetric) ways to the dualistic nature of space and time. This is not the classical interpretation of electromagnetism of the nineteenth century, except for perhaps Clifford. However it is the interpretation necessary to pave the way for a modern unified field theory.

In reality, this duality in the fundamental concept of space and time is or should be reflected in all physical theories of nature that involve differentials in space and/or time. The metric represents an extended line between two centers of mass or gravity, while the non-symmetric (skew- or anti-symmetric) portion represents the center of rotation of motion (Clifford's twist or the later concept of torsion based on his twist) at a point in space. So point and extension differ in the fundamental concept of a rotation. In fact, there are two and only two real points or locations in space that naturally emerge in all of physical reality, the centers of mass (gravity) and rotation.

Reactions of EM and Gravitation to the duality of space



These are real physically discernible points as opposed to the synthetic mathematical concepts of a point as a limited end to a mathematical regression ($\Delta s \rightarrow 0$ in differential geometry and $\Delta t \rightarrow 0$ in calculus) representing a line or other measurable quantity. They are equally represented in quantum theory as point particles and the quantum (boson or vacuum) field of virtual particles. This fundamental duality forms one of the most basic and yet totally unrecognized concepts in all of physics, science and mathematics, as has already been stated and demonstrated.

Isaac Newton also realized something was 'hinky' with the concepts of space and time and spoke directly about the problem as it applied to orbiting planets. For this, he developed his classical thought experiment of the spinning bucket to argue for the existence of an absolute space. However, Newton's concept of absolute space was purely philosophical and had no other mechanical effects except for the existence of a centrifugal force. In the late nineteenth century, the concept of a centrifugal force was erased from physics when vector arguments showed that centripetal forces were the product of cross products between vectors and the need for a balancing centrifugal force to keep the radius of an orbit constant disappeared. Unfortunately, Newton's concept of the absolute space (and time) were based on points which are immeasurable, necessitating his philosophical arguments for its existence, while all of his mechanics was based upon relative and thus measurable distances (extensions) and thus relative space. The mathematical discovery of non-Euclidean and Riemannian spaces was also a blow to the absolute space concept since Newton had equated absolute space with Euclidean geometry. And finally, Ernst Mach used the spinning bucket experiment against Newton's absolute space in his development of what later became known as Mach's principle. All of these philosophical advances, if they were really advances at all instead of ignoring the more deeply rooted fundamental problem of points and a point-space, preceded Einstein's development of special relativity and the supposed death of absolute space by just a few decades. Unfortunately, this development also has the unintended consequence of squashing the concept of spatial dualism and perhaps even helping to bury the point problem in geometry which benefited quantum theory.

Under these circumstances, Newtonian gravity is obviously wrong, although it could be more accurately described as incomplete when it is only expressed as $\boldsymbol{F}=m\boldsymbol{g}$ or its equivalent, since this formulation only includes the more obvious gravity interactions between individual material objects through distances which denote the metric or symmetric nature of space. Newton was well aware of the dualistic nature (point as opposed to extension) of space, but chose to express it differently than it would be described today. Newton's concept of space was the best possible during his day because he only had Euclidean geometry to work with and had no hint or suspicion whatsoever that a more general geometry could be developed. He saw the metric and extended portions of space (and time) as relative and developed his mechanical theories of physical reality upon this basis.

However, he intuitively knew of the second manner in which the concept of space presented itself to physical rendering and developed the notions of absolute (point-based) space and time. His point-based concept of absolute space could not be represented geometrically and used in mechanics, so he equated Euclidean geometry to absolute space by definition rather than observation. Thus Newton only developed the concept of an absolute space or time philosophically as the background or stage upon which relativistic mechanics worked its magic to create our physical world. He tied the two, absolute and relative, together physically in a different more roundabout way by incorporating the concept of absolute space into his calculus of fluxions. Newton's calculus differed from the much later purely mathematical concept of calculus in that the difference in time or 'dt' in the equation always moved forward and could never go to zero since absolute time (the timescale of the universe as a whole) could never disappear completely from physical measurements as implied in the mathematical expression of a limit as $\Delta t \rightarrow 0$.

Newton's mistake with gravity did not go completely unnoticed before the development of relativity at the outset of the twentieth century. Mach developed the notion philosophically in what has become known as Mach's principle that the mass of any object depends on the relative masses and positions of all other material objects in the universe, while Heaviside actually drew a more formal analogy between gravity and electromagnetism. Heaviside instinctively although not necessarily consciously saw the duality of space within the Lorentz equation and reasoned that gravity needed to account for that duality. So he rewrote Newton's gravity theory in the form of

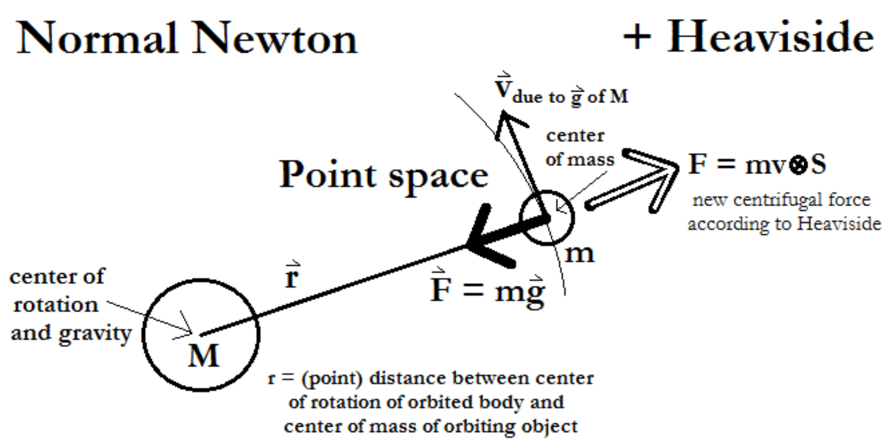
$$\mathbf{F} = \mathbf{m}\mathbf{g} + \mathbf{p} \times \mathbf{S} \, .$$

The quantity $\boldsymbol{p}$ represented the momentum of a body orbiting a central mass and Heaviside correctly described the second term as a true centrifugal force whereby the quantity $\boldsymbol{S}$ represented the gravitational attraction of the rest of the universe on the orbiting body. The rest of the mass in the universe established equilibrium on the orbiting masses such that the orbits remained stable at a fixed distance from the central orbited mass. Thus the second term can be used as a modern mathematical expression of how Mach's Principle contributes to the mechanics of ordinary massive objects.

Within a more modern context the equation can be rewritten as

$$\boldsymbol{F}_{gr} = m\boldsymbol{g} + m\boldsymbol{v} \otimes \boldsymbol{\Gamma} \, .$$

In this case, the small mass represented by 'm' is more distinct than in Heaviside's equation, where it is incorporated into the momentum as an individual fundamental quantity much as in the Heisenberg's uncertainty principle, but the velocity of 'm' as denoted by '$\boldsymbol{v}$' that contributes to its orbit is just the velocity or speed due to the mass m's gravitational attraction to the central body mass 'M' which contributes to or is wrapped up in the vector **g** in the first term.



If the velocity '**v**' goes to zero and the mass 'm' is just falling due toward 'M' due to gravity, then the Heaviside term goes to zero and does not contribute at all to gravity.

In this new form, far more physics can be gleaned from the mathematical formulation than previously suspected. This ultimately means that any orbiting body or any other body with enough speed to surpass its orbital speed and leave its orbit would gain a very small and almost negligible boost to its overall momentum and speed away from the central orbited object due to its attraction to the rest of the universe. The gain in speed would be more pronounced the further the object travels from the original orbited mass and the closer the object gets to the rest of the universe as represented by the variable $\boldsymbol{\Gamma}$. Even in the case of objects that have not attained a true orbit as they accelerate away from a central massive body, their speed at any time would indicate a specific attainable or preferred radius of orbit. So the speed difference between orbit they are passing through and the specified orbit for their actual speed would be a factor in calculating the extra speed gained by the object from its attraction to the rest of the universe as specified by $\boldsymbol{\Gamma}$. This exact effect has been observed by NASA as increases in speed of satellites that cannot be explained by either Newtonian or Einsteinian gravity theory when they are slingshot around planets. The effect has also been observed as two Voyager satellites leave the confines of our solar gravity field.

However, and more importantly, this new form of Newton's gravity seriously implies that the general theory of relativity is incomplete because it only accounts for the metric portion, m$\boldsymbol{g}$, of Newtonian gravity and completely missed the non-symmetric point portion of Newton's gravity. Einstein's general theory of relativity does not completely conform to the dualistic nature of space because it was only designed to geometrically compensate for Newton's **F**=m**g** or only metric extension space, while Maxwell's theory of electromagnetism did, which is why none of the various schemes to unify the two natural forces ever worked. Those who attempted such schemes merely assumed that the anti- or non-symmetric portion of the Einstein gravity tensor represented electromagnetism without ever suspecting that it actually represented a new gravitational term which corresponds or is only analogous to the magnetic portion of Lorentz' equation alone , which will henceforth be called 'gravnetism'.

Einstein's mistake

Einstein introduced his own concept of a dual space as a unitary field, as it was then called, in 1925. This dual field only indirectly corresponded to the extension (metric)/point dualism of space in that the dual portion of the tensor representing the space-time continuum was both Riemannian metric (extension) to explain gravity and affinely connected (point) to represent the torsion at individual points in space that was thought necessary to represent electromagnetism. Once again this configuration represented the right idea for the wrong reason. Einstein merely introduced the non-symmetric tensor rather synthetically from the beginning. The metric portion of the tensor representing Riemannian curvature was neither changed nor directly affected by the non-symmetric tensor that Einstein introduced. This model was thus the product of a dual metric-affine field that treated both fields, which were metrically and affinely connected, respectively, as equal participants in physical reality. Only the symmetric field could be represented by matrices, while the symmetry requirement was relaxed to derive the electromagnetism from the non-symmetric field by a variational principle. And, like his other attempts, this model failed to produce results matching reality so it was dropped by Einstein within a few years in favor of his adoption of the Cartan geometry in 1929.

A group of Russian scientists have since tried to revive the 1929 Einstein-Cartan geometric structure of space-time to describe a new form of gravity based on the concept of a torsion field. The revival of the torsional concept is also related to the efforts of scientists to develop a concept of gravitomagnetism of gravito-electromagnetism (GEM) based on Heaviside's earlier equation. Heaviside only came upon this formulation through an analogy between electromagnetism and gravity rather than any new theoretical insights. All of these scientists have been unknowingly trying to reinterpret gravity in terms of some form of combined point/extension geometry, but they have missed the point of unification by not placing their interpretation of these equations in those terms. They have also missed the second important implication of Maxwell's theory that electromagnetism requires a higher-dimensional embedding space, so all attempts to develop a theory of torsional gravity, from whatever source they started, have retained the notion of an intrinsic curvature of the space-time continuum.

After the failure with his own torsional model, Einstein quickly moved on to try a five-dimensional model using a projective geometry and other five-dimensional arrangements before trying a bi-vector model and finally returning to his non-symmetric model in 1945. Trying all of these different models based on different hypotheses, Einstein's theoretical research became something of a joke within the physics community. Each different attempt was seen as the 'flavor of the month' rather than the honest attempt to deal with the problem of the individual point in a geometrical world that was only based on the metric extension that it was. In other words, there was truly a continuity and specific method behind all of Einstein's numerous unification attempts that other scientists were unable to recognize, which is why the same problem exists today within the quantum world of the standard point-particle model, quantum field, quantum loop, superstring, brane and other models.

After the failure of all of these attempts, Einstein returned to his non-symmetric model and continued to develop this model until his death in 1955. In the meantime, Schrödinger had become impressed with Einstein's 1929 approach as well as Eddington's belief that the affine connection formed the only path to a truly universal differential geometry that could be used to model the space-time continuum. So during the 1940 until the early 1950s, Schrödinger also worked toward the development of a purely affine unification theory. At first he utilized an affinely connected space with both non-symmetric and symmetric characteristics, but slowly adopted Einstein's notion of a fully non-symmetric tensor in combination with the Riemannian metric or symmetric tensor. In essence, he combined the concepts of Cartan's parallel displacement and an anti-symmetric tensor with Eddington's earlier work on the affine connection and derived essentially the same non-symmetric model as Einstein. Coincidentally, another physicist by the name of Saxby published a paper developing very nearly this same model shortly before Schrodinger's 1944 paper on the theoretical model was published. So it might be argued that these physicists were headed in the right direction toward a unification, but again all of their results produced nothing of lasting value for the same reasons as before.

However, one important point of difference remained between Schrodinger and Einstein's theoretical models. Schrödinger discovered that Riemann's metric geometry was derivable from his affinely connected space and thus naturally placed upon the space-time manifold through a simple method of mathematical construction from his curvature tensor and did not need to be introduced from outside. (Schrodinger, 1950) In other words, there was and still remains a fundamental relationship between the metric spatial or symmetric and affinely connected non-symmetric portions of the unified field that Einstein seems to have sensed at some level of his subconscious, but did not directly pursue rather than the other way around. The Riemannian metric thus seemed to be the product of the affinely connected space-time geometry.

This relationship between the two forms of space or geometric manifold corresponds directly to the proposed point/extension duality of physical space itself that is being modeled through unification using the Riemannian geometry. Furthermore, a field equation emerged from Schrödinger's model by using a variational principle that looked a lot like Einstein's general relativistic field equation with the cosmological term that had by then been discarded two decades earlier after Hubble's discovery that the universe was indeed expanding. By simple reasoning, it could therefore be assumed that the mathematical emergence of a Riemannian metric tensor from an affinely connected non-symmetric tensor that yielded the Einstein gravitational equation with the natural emergence of the cosmological term must be geometrically significant for gravity theory rather than unification with electromagnetism.

Indirect support that the non-symmetric portion of the Einstein tensor represented an unknown and unsuspected gravitational (rather than the suspected electromagnetic) connection came from both the calculations of Einstein as well as critics of his non-symmetric theory. For all intents and purposes Einstein's non-symmetric model was successfully refuted by 1953 even though Einstein believed that approach was correct until the very end of his life. Einstein, Callaway and other physicists attempted to calculate the motion of charged particles through the space-time continuum using Einstein's new theory in 1952. After his attempt, Callaway was able to demonstrate that Einstein's model gave the same results no matter how much charge was piled onto the object moving through the unified field and that the Lorentz equation could not be derived from Einstein's unified field equation. Einstein's own calculations showed that it did not matter whether the accelerated object was charged or not and the calculated speeds were far too small or negligible relative to the true motion of charged bodies in electric or magnetic fields. So from this time forward Einstein knew that his theory was a failure, but he did not give up hope.

He had published a new edition of his book *The Meaning of Relativity* in 1951, before the calculations were made and his theory failed, but he then wrote an even newer edition of the book for publication. It was published in 1956, shortly after he died. In this last edition Einstein took full responsibility for the failure of his theory, but again stated that he believed he was on the right track and that a unified field theory of the type that he sought was the only answer to unification. He penned the final version of his non-symmetric theory in the first appendix and spoke about the theory in the second appendix. He also admitted that if his non-symmetric approach failed, there were other related methods that might work.

> GENERAL REMARKS
>
> A. In my opinion the theory presented here is the logically simplest relativistic field theory which is at all possible. But this does not mean that nature might not obey a more complex field theory. More complex field theories have frequently been proposed. They may be classified according to the following characteristic features:
>
> (*a*) Increase of the number of dimensions of the continuum. In this case one must explain why the continuum is *apparently* restricted to four dimensions.
>
> (*b*) Introduction of fields of different kind (e.g. a vector field) in addition to the displacement field and its correlated tensor field g_{ik} (or $\mathfrak{g}_{ik}$).
>
> (*c*) Introduction of field equations of higher order (of differentiation).
>
> In my view, such more complicated systems and their combinations should be considered only if there exist physical empirical reasons to do so.

> B. A field theory is not yet completely determined by the system of field equations. Should one admit the appearance of singularities? Should one postulate boundary conditions? As to the first question, it is my opinion that singularities must be excluded. It does not seem reasonable to me to introduce into a continuum theory points (or lines, etc.) for which the field equations do not hold. Moreover, the introduction of singularities is equivalent to postulating boundary conditions (which are arbitrary from the point of view of the field equations) on 'surfaces' which closely surround the singularities. Without such a postulate the theory is much too vague. In my opinion the answer to the second question is that the postulation of boundary conditions is indispensable. I shall demonstrate this by an elementary example. One can compare the postulation of a potential of the form $\varphi=\sum m/r$ with the statement that outside the mass points (in three dimensions) the equation $\Delta\varphi=0$ is satisfied. But if one does not add the boundary condition that φ vanish (or remain finite) at infinity, then there exist solutions that are entire functions of the x (e.g. x^2_1 - ½(x_2^2 +x_3^2)) and become infinite at infinity. Such fields can only be excluded by postulating a boundary condition in case the space is an 'open' one.
>
> C. Is it conceivable that a field theory permits one to understand the atomistic and quantum structure of reality? Almost everybody will answer this question with 'no'. But I believe that at the present time nobody knows anything reliable about it. This is so because we cannot judge in what manner and how strongly the exclusion of singularities reduces the manifold of solutions. We do not possess any method at all to derive systematically solutions that are free of singularities. Approximation methods are of no avail since one never knows whether or not there exists to a particular approximate solution an exact solution *free of singularities*. For this reason we cannot at present compare the content of a nonlinear field theory with experience. Only a significant progress in the mathematical methods can help here. At the present time the opinion prevails that a field theory must first, by 'quantization', be transformed into a statistical theory of field probabilities according to more or less established rules. I see in this method only an attempt to describe relationships of an essentially nonlinear character by linear methods.
>
> D. One can give good reasons why reality cannot at all be represented by a continuous field. From the quantum phenomena it appears to follow with certainty that a finite system [appendix ii 169] of finite energy can be completely described by a finite set of numbers (quantum numbers). This does not seem to be in accordance with a continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory. [Einstein, 1956, 168-170]

Unfortunately, Einstein never realized how close to the unified field theory he had come. In a very real sense, the answer to the unified field theory mystery was to combine the different methods into a single theory as had been done by Schrödinger with the affinely connected spaces and the concept of parallel transport. The key to that tactic was even evident in the calculated failures of his non-symmetric theory.

A careful look at Einstein's and Callaway's concluding comments following their respective calculations indicate the answer to solving the puzzle of the unified field as do Schrödinger's success deriving the metric field or the symmetric tensor from the affinely connected non-symmetric tensor. Schrödinger's work clearly demonstrated that a combination of point and extension geometries into a single unified concept was the correct way to treat the space-time continuum and develop the unified field theory. Since he was able to use the variational principle to derive the correction factor (a purely gravitational concept) that Einstein had earlier adopted, dropped and called the cosmological factor without reference to electromagnetism suggested that his model made no reference to electromagnetism. Schrödinger's theoretical model was only about gravitation and suggested a second unsuspected from of gravity emerged form or component the equations. Einstein and Callaway's work demonstrated the same thing.

Callaway could not derive the Lorentz electromagnetic equation from the non-symmetric model because the non-symmetric model had nothing to do with electromagnetism, which should have been evident when Callaway determined that piling more electric charge on the moving object did not change its motion through the field, *i.e.* the extent of motion was independent of the amount of charge piled on the moving object. Einstein's own calculations confirmed this conclusion. Einstein noted that the motion of the accelerated body did not change whether the moving body was or was not charged. Charging the body was irrelevant to his theoretical model as it should be to a purely gravitational model, while the fact that his calculated speeds were negligibly small indicated the action, interaction or effect of a weak natural force like gravity rather than a strong natural force such as electromagnetism.

In other words, all indications were that the non-symmetric portion of the unified field was about a secondary effect of gravity and not about electromagnetism, but no one came to this conclusion. What Einstein, Callaway and others really calculated in 1952 was the dark matter and dark energy effects of the symmetric/non-symmetric gravitational field, but not knowing about such matters (which had not yet been 'discovered' even though it has been observed as far back as the 1930s), they allowed their minds (biases and prejudices) to trick them into thinking that the theory was completely wrong rather than declaring that a new gravitational effect had been predicted by the theory. The non-symmetric portion of the Einstein tensor simply revealed the existence of a mathematical term representing a secondary effect of normal gravity, which can be called gravnetism, but no one recognized this fact and thought the theoretical model a failure because it did not yield the desired results for electromagnetism, not could it.

The whole problem boils down to the simple fact that Einstein and the others working toward a unified field theory were working with an incomplete geometrical picture of space and manifolds that did not include both point and extension (metric) as connected geometric equally contributing factors. Many scientists and mathematicians had seen that the geometry was incomplete and tried to take advantage of that incompleteness (developing non-Riemannian geometries, tangent spaces, pseudo-Riemannian geometries, affinely connected spaces and so on), but no one actually tried to attack the real central problem of how discrete points could also be represented in continuously extended physical space. In the end the answer to this dilemma is that **physical space is dualistic**, *i.e.* physical space requires both extension as well as point expression for any force that acts within it. Electromagnetism already expresses both, but gravity is only expressed by its extension (metric) in space. Therefore the gravity theories or both Newton and Einstein are incomplete and the two forces cannot be unified by using non- or anti-symmetric tensor components for electromagnetism as Einstein and others attempted.

John Moffat came to the correct conclusion in 1979, that the non-symmetric portion of the Einstein tensor has absolutely nothing to do with electromagnetism because it represented a purely secondary effect of gravity due to the dualistic nature of space itself, but he reached the same conclusion from a completely different direction. Therefore he has been unable to correctly develop a unified field theory from his correct approach. He is still missing one element. The only others who presently seem interested in this work are historians associated with the Einstein Papers project such as Tilman Sauer, another historian by the name of Hubert Gönner, the American physicist James Shifflett (PhD dissertation, 2005) and an Australian physicist by the name of Vu B. Ho. But each of them is still looking at only one approach or problem without seeing the whole picture. The Einstein metric tensor of classical general relativity should have two parts – symmetric and non-symmetric – due to the dualistic nature of space, and it cannot be unified with electromagnetism until both parts of the gravitational field are expressed as are both parts of the electromagnetic filed. The symmetric portion yields space-time curvature to explain Newtonian gravity while the non-symmetric portion which was thought (wrongly) by Einstein and others to explain electromagnetism actually predicts what is now wrongly called dark matter (it is not dark because it is the normal baryonic matter that causes normal gravity) and dark energy (which is really just gravitational or gravnetic vector potential).

And therein a suggestion to the correct solution for unifying physics presents itself. The gravnetic (gravitational) vector potential and the magnetic vector potential are not intrinsic to the curvature at discrete points in space (in the non-Riemannian-like and tangent space-like fashion), but rather imply that curvature must be extrinsic within a higher-dimensional embedding manifold or space-like structure. That hyperspatial structure, or at least an over-restricted (abbreviated) example of it, already exists in the Kaluza (1921) five-dimensional model of the space-time continuum. However, the five-dimensional Kaluza model could not be applied to unification with complete success until the issue of the physical characteristics of the correct geometrical structures of the embedded space and its embedding manifold has been resolved.

The extra-dimensional theory was always implied by the Riemannian geometry used by Einstein, but it seems no one paid attention to it. Perhaps scientists thought of time as the embedding dimension for the three-dimensional space of experience, but there is little to no mention of that likelihood in the scientific literature that is available. An extra dimension is also implied by Maxwell's electromagnetic theory. The del function and the del cross operation imply a fourth dimension of space as well as the magnetic vector **A**. Clifford, a friend of Maxwell's as well as the translator and popularizer of Riemann's differential surface geometry to the English speaking world, attempted to derive a four-dimensional electromagnetic theory based on Maxwell's earliest work and the implications of Clifford's mathematical researches seem like a harbinger behind all modern unified field theories, whether based on the quantum or on relativity. Clifford's work has either directly or indirectly influenced all of modern physics.

III. A new dimension of physics

Kaluza's five-dimensional unification

Indications that a higher-dimensional space is a physical necessity for the advancement of theoretical physics were quite evident well before general relativity was first developed. The original development and spread of Riemannian and the non-Euclidean geometries of course implied that curvature required a higher embedding dimension and this was exactly how the new geometries were interpreted well before relativity theory was thought to have introduced space-time curvature into physics. The first impression of nearly everyone during the nineteenth century was that the new non-Euclidean and Riemannian geometries were physically possible, possibly represented real physical space and that they required a higher dimension. By the 1880s, various astronomers had even attempted to use parallax measurements to verify and measure the curvature of space. In other words, the curvature of physical space was real and extrinsic in today's scientific language. Moreover, the newly developed electromagnetic theory of Maxwell also implied a higher-dimensional space, not only because it was first formulated in quaternions which many interpreted by some as four-dimensional but even after the theory was rewritten in vector form.

This type of structure for the embedding space (Riemann's manifold concept) is clearly implied in the classical electromagnetic concept of the magnetic vector potential. The vector potential is defined as **B**=del cross (the 'del cross' operation is also called a curl) **A**, where **B** is the magnetic field strength and **A** is the vector potential. The del function or operator is defined as

$$\nabla = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}).$$

As this equation indicates, the del function operates in all three dimensions of space (represented as x, y and z) simultaneously, so its cross product with the vector potential **A** yields the magnetic field **B** in three-dimensional space. Therefore **A** must be perpendicular to all three dimensions of normal space simultaneously and thus extended in the fourth dimension of space. Furthermore, del dot **A** equals zero, which also confirms that **A** cannot exist in three-dimensional space since the 'dot' product between vectors yields a cosine of the angle between the vectors. Since the dot product of the **A** and **B** vectors is zero, an angle to take the cosine of cannot be established between them in three-dimensional space, thus **A** can only exist in a perpendicular fourth dimension of physical space.

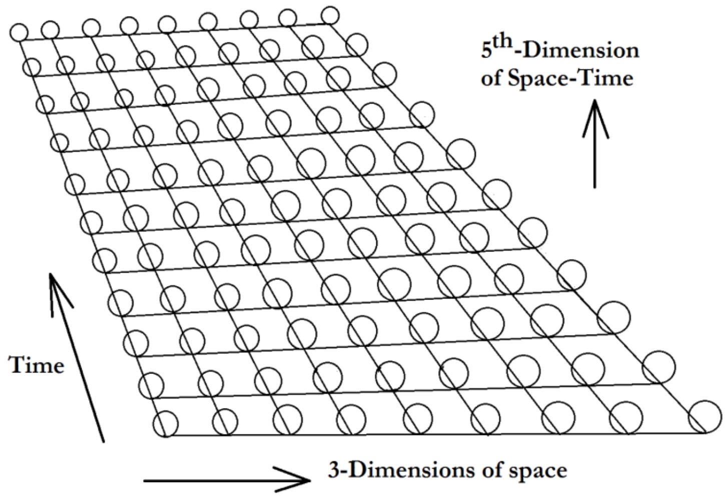
The cross product for normal vectors in three-dimensional space can be visualized or graphed as one vector with a common starting point rotating through an angle into another vector (thus called the curl). So **A** cross **B** would have the value of AB sin of the angle between them. However, del cross either **A** or **B** cannot be so easily visualized since the derivatives in three-dimensional space indicate a point in space. There is absolutely no way that a dimensionless geometrical point can turn into or 'curl' into another dimensionless geometrical point simultaneously in all three dimensions within a given three-dimensional space even though that is exactly what is implied when the del cross operation is administered to real physical vectors. That is why Clifford developed the concept of a 'twist' and associated the 'twist' with real points in physical space in order to explain magnetic induction. The solution to this enigma is that the twist or curl projects the vector into along the perpendicular axis of the higher dimension, which could be visualized as a sphere imploding into itself or rather twisting and curling into its dimensionless point center simultaneously along all three-dimensions of normal physical space.

Isolating and even measuring the magnetic vector potential **A** has been impossible, but the **A** vector is an integral and important piece of electromagnetic theory without which electromagnetic theory would fail. So the concept is accepted by all scientists as merely a mathematical artifact of electromagnetic theory without any physical possibility of measuring it. How could that be possible? The magnetic potential vector **A** does exist and has been experimentally verified. The Aharanov-Bohm and similar experiments have demonstrated that **A** is a real measurable quantity even if it is impossible to isolate and measure within three-dimensional space. All of the measuring instruments of science are materially three-dimensional so science should not be able to directly measure a strictly four-dimensional object such as the magnetic vector potential **A** and so the question of the physical reality of **A** has a simple although higher-dimensional solution. Furthermore, this means that even classical electromagnetic theory demonstrates a problem with associating physical reality with individual infinitesimal points of space since the point is implied as being projectable or extendable into the higher embedding dimension of space. If, on the other hand, the curl of the magnetic potential field **B** is taken, such that del cross B equals aero, then B must exist totally, completely and only in the normal three-dimensional space of experience.

knowing this fact, the physical role of point geometries was first noted by William Kingdon Clifford in the 1870s. Clifford may be better known for 'anticipating' general relativity by stating that matter is nothing but curved space and the motion of matter is nothing but variations in that curvature, but when he developed his theory of matter, he did not use Riemannian geometry as Einstein later did. He shied away from gravity, instead working toward a theory of magnetic induction based on an hyperspatial point geometry of his own design using twists, biquaternions and screws (a twist along a linear displacement) with exhibited torsion. His biquaternions represented magnetic vector potentials extended in the fourth dimension of space. Clifford's theoretical work is all but forgotten today, but it did influence the further development of geometry by Felix Klein in 1890, who published his version of Clifford's geometry after Clifford's early death, and Élie Cartan who developed a point geometry of spinors based on Clifford's efforts and later developed his own physical unification model. Cartan's geometry was then used by Einstein in his 1929 attempt to unify general relativity and electromagnetic theory, utilizing a concept called distant parallelism, even though he and others completely ignored Clifford's original notion that the twist-points in space required a higher-dimensional embedding space or manifold. The same is true for the theoretical work of Roger Penrose, who developed his concept of twistors a good deal later based on Clifford's original work and idea of a twist that characterized each geometric point in physical space.

Given these historical circumstances, it is little wonder that Kaluza is came up with the idea to unify Einstein's general relativity theory with Maxwell's electromagnetic theory by assuming an extrinsic curvature in an embedding fifth dimension of space-time. A more interesting question would be 'why was he the only person to do so?' This question is all the more significant since there is ample evidence that many scientists already interpreted general relativity as five-dimensional even before attempts were made to unify gravity and electromagnetism. Eddington, Ludwig Silberstein, Willem de Sitter and others originally interpreted Einstein's space-time continuum embedded in a higher dimension of space, *i.e.* Einstein's curvature was intrinsic. (Beichler, Twist 'til we tear the house down) Kasner first proved that Einstein's theory could not account for planets orbiting the sun in a five-dimensional flat space and then published papers utilizing Einstein's curvature to explain the orbits of planets around the sun embedded in a six-dimensional flat space, the first coming a month before Kaluza published his only paper utilizing a five-dimensional space-time framework.

As originally expressed, Kaluza's theory was little more than the barest attempt to justify and utilize a higher dimension of space if not a mathematical gimmick used to obtain the extra parameters and equations necessary to express a combined field and describe already known electromagnetic phenomena. No one will ever know the extent to which Kaluza believed in the physical reality of the higher spatial dimension since he neither gave it nor specified any physical characteristics by which it could be distinguished. In other words, he assumed that each point in Einstein's curved space-time continuum was extended a very short distance into the higher embedding dimension to form what he called A-lines. All A-lines were of equal length and each A-line exited three-dimensional space and returned to that some point in three-dimensional space to form a closed loop.



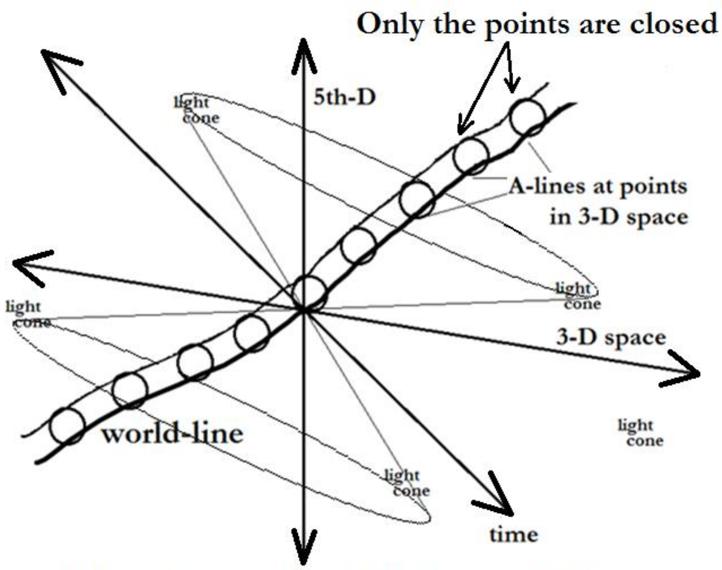
Assume each intersection is a dimensionless point in 3-D space and there is no space between points. At each dimensionless point in 4-D space-time a perpendicular line called an A-line extends into the 5th-D and closes back into the point in 4-D space-time. Each A-line or loop is equal and constant length. Since all of the loops are are of equal size in the 5th-D and parallel to each other, they form a cylinder (A-cylindricity) of sorts in the higher dimension. The 5th-D of space-time is not itself closed with respect to 4-D space-time, but each individual point is closed in the higher dimension with respect to 4-D space-time.

Taken together, these two characteristics vastly limited his mathematical model, but they also allowed him to make mathematical calculations without associating the higher dimension with any specific physical characteristics. These two conditions were interpreted as inter-dependent or as one and together called the 'cylindrical condition'. They did not really form a cylinder in higher space because each A-line was only connected to other A-lines to form a continuous surface through the continuity of three-dimensional space where the A-lines originated and terminated, although such a cylinder could be imagined.

Given these conditions, the mathematical procedures developed by Kaluza yielded a quantity that resembled electromagnetic potential φ, to which Kaluza made an analogous equivalence. This allowed him to introduce a tensor form of Maxwell's equations. However, Kaluza made the same mistake as all others who attempted unification. He closed his extensions in the higher space, the A-lines, individually (point-by-point), but did not close the higher space as a whole. In other words, he made no three-dimensional connection between the extensions into the higher dimension other than their normal three-dimensional continuity. By so doing he extended the individual points into the higher dimension, but did not extend the whole of the three-dimensional metric surface with its continuity into the higher embedding dimension just as those who developed non-Riemannian geometries did not connect the non-Riemannian geometries of individual points to each other to generate the intrinsic Riemannian metric of space-time. Continuity was merely a three-dimensional space or four-dimensional space-time characteristic for everyone who originally developed unification geometries.

Kaluza's limiting and restrictive mathematical methods rendered his five-dimensional model incapable of yielding predictions that could verify the model. Unification via Kaluza's theory was successful in what it attempted, but vastly minimized and overly limited in its scope and application since it merely duplicated Maxwell's theory using some questionable mathematical techniques. Kaluza only extended the individual points in the four-dimensional space-time continuum into the embedding five-dimensional manifold without saying anything about either that manifold (its geometry) or how the whole metric could be extended into the higher dimension. Kaluza's model was merely another 'tangent space' as were the non-Riemannian geometries except that he used the extra embedding dimension and his extended geometry had no discernible geometrical characteristics while the non-Riemannian attempts foreswore the implied (even suggested) embedding dimension for the sake of internal (intrinsic) characteristics that could possibly yield a more general overall geometry.

So Kaluza did not adequately tie or connect the points to each other to form a continuous extension in normal four-dimensional space-time while every A-line closed loop into the higher embedding dimension was completely independent of any portion of the other A-lines extended in the higher-dimensional embedding dimension.

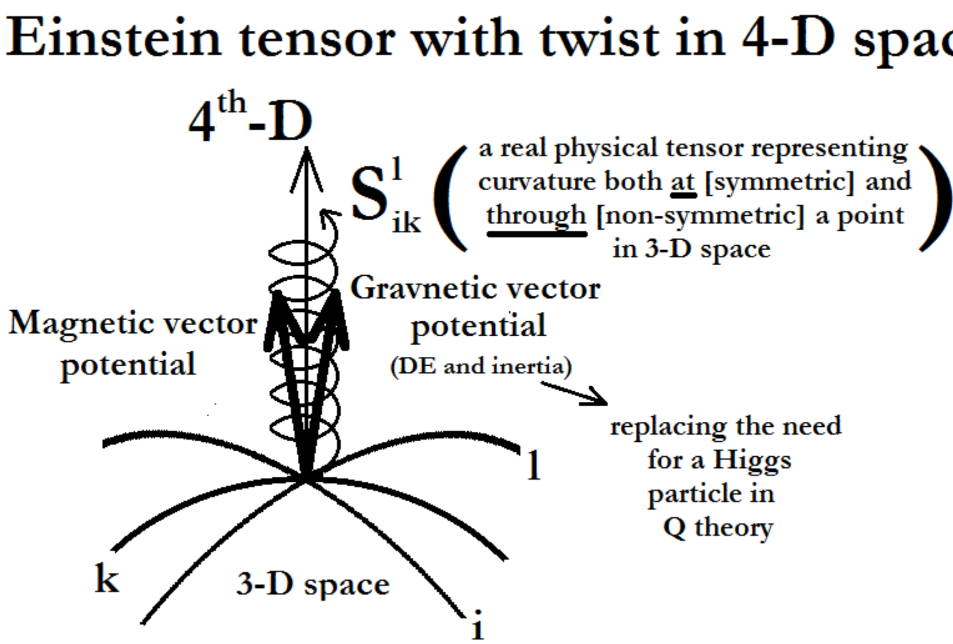


The world-line of an event in the light cone of 4-D space-time is an infinitesimally miniscule cylinder in the 5th-D of space-time

Picturing the A-lines in this manner implies a simulated but by no means real three-dimensional continuity all along the A-lines, even outside of the curved three-dimensional space. So the five-dimensional Kaluza model also suffered a similar incompleteness as the other unified field theories that retained the intrinsic curvature of the space-time continuum. Nor did he attempt or even recognize the possibility that continuity, even in three-dimensional space, had not been completely demonstrated by mathematicians and thus only took continuity in teh normal four-dimensional space-time for granted as an assumption.

Once Kaluza's mathematical system was properly set-up, he applied a cut-transformation to the equations, which yielded the electromagnetic field, and a four-transformation, which yielded the Einstein gravity field. Although Kaluza was able to duplicate Maxwell's formulas (some scientists thought even this duplication was artificial) in this manner, the geometry he used (based on the cylindrical condition) was too restrictive and did not allow any further deductions or testable predictions to be made about the nature and consequences of the higher dimension of space. Since his theoretical model yielded no prediction to be tested, his theory was scientifically fallow (barren) for the most part and for several decades was nothing more than a curiosity within the scientific community. So the real problem remained how points and extensions in physical space could be related to one another in spite of Kaluza's partially successful unification model. So Kaluza only developed another piece of the puzzle even though his theory was successful in a very limited manner.

In reality, the gravnetic (gravitational) vector potential and the magnetic vector potential **A** are not intrinsic to the curvature at discrete points in space (in the non-Riemannian-like and tangent space-like fashion), but rather imply that curvature must be extrinsic in a higher dimensional embedding manifold or space-like structure, thus implying the necessity of a five-dimensional embedding manifold.



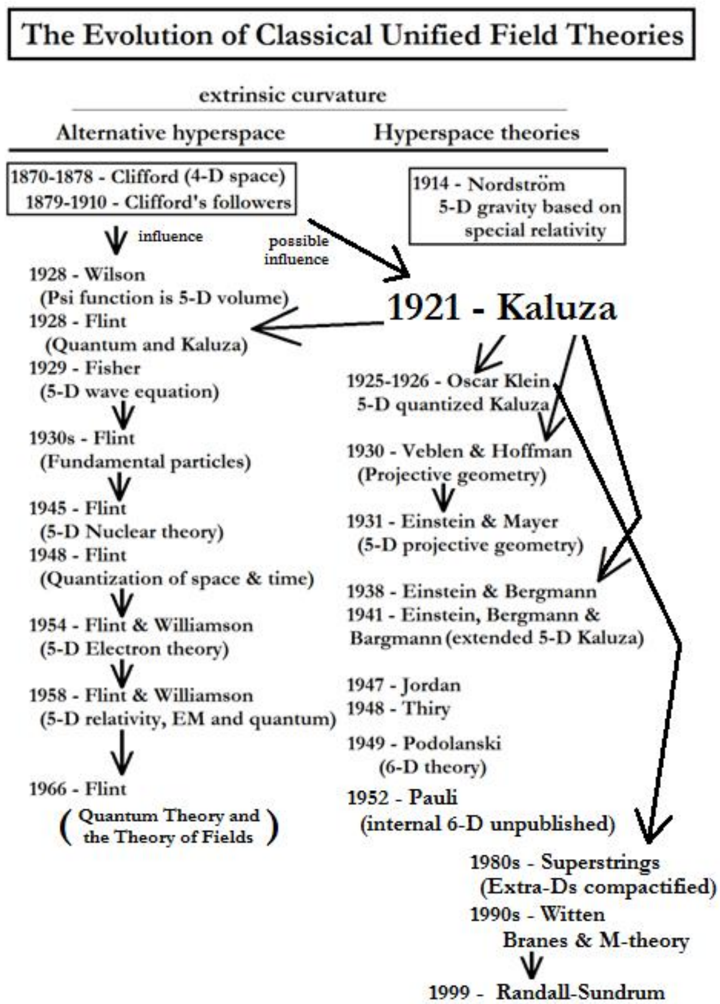
Furthermore, if the non- or anti-symmetric tensor represents a secondary and unsuspected effect of gravity due to the dualistic nature of space itself, how can electromagnetism and gravity be unified into a single theory? Actually this structure simplifies the unification because gravity-gravnetism is now symmetric with electricity-magnetism. So despite the fact that everything points to a higher fourth dimension of space as well as the (point/extension) duality of space, no one has either attempted or even suspected combining the two methods to unify physics. Yet by recognizing the promise and validity of the hyperspatial model, some scientists have seen it instead as a way to unify gravity and the quantum.

Oskar Klein was one of the few scientists to take Kaluza's model seriously and interpreted it as it was, shortcomings and all, to unify relativity and teh quantum. In 1926, 1927 and later, he published papers extending by modification Kaluza's five-dimensional model to include the quantum. He noticed that the A-line loops formed a periodicity that could be quantized and thus the Kaluza-Klein model was born. At nearly the same time an independent paper was published by Wilson in England that sought to incorporate the quantum with a higher spatial dimension. Wilson likened the Schrödinger psi function to a volume in five-dimensional space and upon this hypothesis he was able to derive the Klein-Gordon equation of quantum theory. The extent to which Wilson was aware of Kaluza's original work at the time is not known, but he was quickly apprised of both Kaluza and Klein's work before his paper was published, so he referred to them in a footnote in his published paper.

Since he was unfamiliar with Kaluza's approach, Wilson took a completely different track than Klein's quantum model. A number of other physicists associated with Wilson followed his lead and began to work on further developing the five-dimensional model, whereas Klein was quickly talked into temporarily dropping his five-dimensional approach by Bohr and others at the seventh Solvay Conference. At this same conference, De Broglie was dissuaded from following up on his 'pilot wave' theory (also called the theory of the double solution). He did no further work on the theory until the early 1950s at the behest of David Bohm and Jean-Pierre Vigier. However, the conference was far more famous for the fact that Heisenberg introduced his uncertainty principle and the new quantum mechanics, where he and others were able to demonstrate a clear and precise equivalence between Schrödinger's wave function and teh probabilities implied by the uncertainty principle. Bohr and Heisenberg then introduced the Copenhagen Interpretation of the quantum whereby the final nature of physical reality was unknowable and yielded physics itself yielded probabilistic results, i.e. nature was discrete and indeterministic according to both quantum and wave mechanics.

This conference marked the end of the classical period of the quantum theory and the beginning of the quantum mechanical or indeterministic period of the quantum even though quantum field concepts were in the air at the time. It also marked a number of splits in science, the most notorious of which was the severing of Einstein from any new theoretical advances in the quantum theory and his ensuing isolation within the scientific community. Einstein fought Bohr and Heisenberg to a draw (although many believe that Einstein was beaten by Bohr in the debate) concerning the interpretation of the quantum, while the Copenhagen Interpretation became the unquestionable new standard for physics.

Meanwhile, Flint was chief among the followers of the five-dimensional hypothesis in England and a school that lasted several decades was formed around his research. Flint was Wilson's student in the beginning, but quickly became his partner in developing the five-dimensional unified field theory. All in all, this group of Wilson, Flint and their students produced and published over forty articles in British scientific journals over the next four decades, but their work has seemingly been forgotten and lost to history. Yet Flint probably came closer than any other physicist to unifying physics, except for Einstein.



Flint alone published nearly forty papers on his five-dimensional unification model which remain all but hidden in plain sight from modern scientists and academics. Over the course of time, Flint incorporated every advance in both relativity theory and quantum theory into his model, which rendered his approach to unification as well as his method of achieving that goal vastly different from Einstein's and all other attempts at unification. Flint's final publication on unification came in 1966 with his book *The quantum equation and the theory of fields* shortly after he retired from his teaching position.

Wilson and Flint obviously noticed the value of the five-dimensional model for incorporating the quantum into relativity very early on in the development of the quantum theory. Wilson even stated that doing so was the only way forward for physics. Yet this strategy for advancing physics has been downplayed by almost all other scientists over the past century. How could this have been? Quite simply, this group of English scientists did not accept the Copenhagen Interpretation of the quantum and quantum mechanics. So their work implied that the quantum and relativity could be easily unified if the Copenhagen Interpretation, which puts up synthetic barriers between the quantum and relativity (discrete versus continuity and determinism versus indeterminism) that cannot be easily overcome as well as discourages attempts to do so. So if the Copenhagen Interpretation is disregarded, the unification of the quantum and relativity should not be that difficult although it should be made within a five-dimensional framework of space-time.

Wilson and Flint's fundamental interpretation of the quantum itself came from another direction entirely. At nearly the same time as Wilson was publishing his early papers on electronic orbits in the atom, Tetrode was also publishing (1912-1925) fundamental papers on the quantum theory. Tetrode clearly demonstrated that Planck's constant could be introduced into the thermodynamical concept of entropy of an ideal gas if one considers that the number (**N**) of statistical states or cells in a volume of gas is proportional to $\mathbf{h}^{3N}$ in his very first paper. This allowed him to quantize molecular motion rather than the supposedly discrete blocks out of which reality was constructed, which means that Planck's constant was no longer tied to just the oscillatory or vibrational states of elementary particles. He later applied this concept to classical entropy formulas such as the heat capacity of matter. Wilson, who was working on the concept of photon emission from ideal gases would have been greatly impressed by this work, but more importantly Tetrode's work treated Planck's constant and the notion of the quantum in a more classical and deterministic manner than later became the standard with the Copenhagen Interpretation and Heisenberg's uncertainty principle. Nature and the physical world were not indeterministic as in the later Copenhagen Interpretation, but instead quantized states were necessary in the description of nature because science is unable to precisely determine or measure positions, speeds and momentums simultaneously for all particles in any given system

In 1922, Tetrode went even further and turned his own attentions in a whole new direction by introducing a radically new approach to the question of quantum mechanics. In a newly published paper, he developed the concept of a time-symmetrical 'action-at-a-distance' theory based on Maxwell's electromagnetism did not distinguish between waves traveling backward and forward in time. A few years later, in 1928, Tetrode turned away from the time-symmetric model and instead began to develop a general relativistic quantum model of electrons. This work did contain a relativistic generalization of Paul Dirac's theories, which were quite a hot topic among quantum physicists at that time. The energy-impulse tensor that Tetrode developed within this context did, however, survive and is used today under the name Tetrode-tensor. So it can be easily demonstrated that there is more to quantum theory than expressed by the opinions of Bohr and Heisenberg and their CI. This whole exercise implies that the fundamental nature of the quantum and its relationship to geometry and space-time has never been truly explored, which returns us to a closer look at Riemann's geometry as it was originally developed and answer the question if any part of Riemannian geometry implies or leads to a solution of the point/extension dualism.

In the end, the significance of the five-dimensional space-time structure is every bit as important as the geometric duality of space (and time) for developing a true unified field theory. Yet this is also a historical problem dating back into the science and mathematics of both the near and distant past. The problem in the past to which all other attempted unified field theories have succumbed was one of 'mistakenly misrepresenting' these two approaches (point and extension) and/or assuming that they could replace one another in that solving one incorporated a solution for the other. This was the reasoning behind the approach of the intrinsic theorists. Quite literally, academics of the past, like those of the present, did not understand the depth of the problems in their considerations of mathematics and science, which is especially true in the case of the point/extension duality. Science has to take into consideration that our concepts of both space and time have no dimensions except for the fact that we measure and/or determine them by the relative positions of material bodies, their constancy and various changes in them and their positions in space and time. So the concepts of constancy (non-change) and change from another level of the basic duality with which science and mathematics must deal. Constancy (non-changing) is directly related to the continuity concept which manifests in extension and change as related to location and thus individual geometrical points in space and time. As for the concepts of point and extension, Riemann noted them both, but only partially addressed the nature of their duality.

Riemannian geometry

In his original paper on generalized geometry, Riemann clearly noted the difference between a point-element and a metric-element, but he chose only to base his differential system of surfaces on the metric concept. Yet he could not ignore the importance of the point-element for his or any other geometry so he considered a limiting approach to expressing his metric-element in the form

$$\mathbf{ds} = \sqrt{(\mathbf{dx})^2},$$

and then allowing it to approach zero extension (without actually equaling zero)

$$\mathbf{ds} \rightarrow \mathbf{0}.$$

This means that the line-element is shrunk continuously down to but not quite equal to zero, *i.e.* it approaches zero in value or length. In other words, Riemann looked at and considered an infinitesimal from the point-of-view of shrinking his extended surface (of any number 'n' of dimensions) as small as possible to approximate a true point element, but only to approximate it. It is exactly this property of the Riemannian geometry that the developers of non-Riemannian geometries took into account when they placed other geometries within the actual infinitesimal points along the curved surface of the space-time continuum.

However, Riemann was well aware of this deficiency in his geometrical approach to space. He must have thought this strategy sufficient, but not without qualifications. He actually stated so in his thesis.

> Every system of points where the function has a constant value, forms then a continuous manifoldness of fewer dimensions than the given one. These manifoldnesses pass over continuously into one another as the function changes; we may therefore assume that out of one of them the others proceed, and speaking generally this may occur in such a way that each point passes over into a definite point of the other; the cases of exception (the study of which is important) may here be left unconsidered.

Although this particular quote is taken from Clifford's translation of Riemann's paper, we can assume that the sentiment expressed was Riemann's and not due to Clifford, although Clifford still used this idea as a working point in developing his own geometry in 1873 and 1876. Riemann only assumed that individual geometrically dimensionless points in his surface could pass from point-to-point to develop the extended surface without proving that to be the fact. It is exactly this notion or assumption that is missing from geometry, whether physical or purely mathematical, from Euclid until the present day. To his credit Riemann noted the possibility of exceptions to his rules, but chose not to consider them, once again opening the question of geometric role of true points for consideration by others. The incompleteness of Riemannian geometry was thus planned and thus neither accidental nor oversight.

Ultimately, the question and the problem of points itself goes all the way back to Euclidean geometry as portrayed in Euclid's *Elements* (~300BCE). Non-Euclidean geometries were first developed to overcome a suspected flaw in Euclid's geometry. It was claimed by those who later sought to correct and complete geometry that the parallel axiom was the only statement, proposition or axiom in the *Elements* that was not proven within the *Elements*. Yet even this statement is untrue. It was never proven that infinitesimal points could be joined together in any logical structure to 'build' or 'construct' an extended line, surface of volume or any kind. Geometers just assumed that this was possible and let ti go at that. This was the start of the point problem which is still alive, under many different disguises and names, in modern mathematics, be it the number line in arithmetic or the extended line in geometry.

The non-Euclidean geometries, including Riemann's, were developed to overcome a shortcoming in Euclid's geometry called alternately the parallel axiom or the parallel postulate. Euclid stated or rather assumed as a postulate that parallel lines (equidistant straight lines) would always remain equidistant unto forever and infinity. So, the parallel problem was really a problem dealing with the ambiguous nature of infinity or the infinitely large, while the unsuspected and undocumented point problem dealt with the ambiguity of the infinitesimal or infinitely small. In some respects then they are just the same problem, although solving one says nothing about the other. This then is the same problem faced by the Heisenberg Uncertainty Principle in physics, which is merely one (although the most obvious since it is open to such a wide variety of interpretations) manifestation of the physical side of the problem. Later attempts to prove this postulate rendered discrepancies of parallel lines that either converged (Riemannian or spherical geometry) or diverged (Lobachevskian, Bolyaian or hyperbolic geometry) at infinity as opposed to Euclidean (flat geometry) where they remained forever equidistant.

Euclid also assumed that every extension, no matter how large or how small, consisted of an infinite number of points and any break in a line or extension would occur at an infinitesimal point. If this were true and if limiting the line-element to approach zero as an approximation to zero actually yielded zero value, then the opposite would also be true: That any extension or line-element could be constructed or built up by adding points or point-elements together. Yet this last idea has never been demonstrated and possibly never even attempted, so it creates a rather large discrepancy in all geometrical systems. It remains, even today, as another unproven postulate or assumption in Euclid's geometry whether it has ever been stated and recognized as such, or not.

Riemann seems to have been aware of this discrepancy in mathematical logic or at least something like it at some level of consciousness (like many others after general relativity), although he did not attempt to tackle the problem and provide a proof for the hypothesis. Instead, Riemann looked at geometry as a physical science and stated that

> Hereby the determination of position in the given manifoldness is reduced to a determination of quantity and to a determination of position in manifoldness of less dimensions. It is now easy to show that this manifoldness has n - 1 dimensions when the given manifold is n-ply extended. By repeating then this operation n times, the determination of position in an n-ply extended manifoldness is reduced to n determinations of quantity, and therefore the determination of position in a given manifoldness is reduced to a finite number of determinations of quantity when this is possible. There are manifoldnesses in which the determination of position requires not a finite number, but either an endless series

> or a continuous manifoldness of determinations of quantity. Such manifoldnesses are, for example, the possible determinations of a function for a given region, the possible shapes of a solid figure, &c.

Riemann was relating the position of an individual discrete point, which required an extended or *n*+1-dimensional embedding space or manifold over which a mathematical function (based on a continuity principle) could be applied, to the physical measurement of an extended surface. In other words, Riemann considered a physical or semi-physical geometry in which continuity was the rule and thus truly discrete (marked by complete breaks in continuity) points were not possible.

But Riemann also knew that the measurement of discrete points could not be accomplished in physical space, so he spoke instead of a quanta or smallest measurement that could be taken or physically made.

> Definite portions of a manifoldness, distinguished by a mark or by a boundary, are called Quanta. Their comparison with regard to quantity is accomplished in the case of discrete magnitudes by counting, in the case of continuous magnitudes by measuring. Measure consists in the superposition of the magnitudes to be compared; it therefore requires a means of using one magnitude as the standard for another. In the absence of this, two magnitudes can only be compared when one is a part of the other; in which case also we can only determine the more or less and not the how much. The researches which can in this case be instituted about them form a general division of the science of magnitude in which magnitudes are regarded not as existing independently of position and not as expressible in terms of a unit, but as regions in a manifoldness. Such researches have become a necessity for many parts of mathematics, e.g., for the treatment of many-valued analytical functions; …

So, while Riemann spoke of mathematically pure concepts, he was actually thinking about physically real and measurable objects in space that could not be reduced to points because they were really extended and extended bodies have specific measurable or otherwise detectable boundaries.

Within this context, Riemann strongly implied if not directly noted that the smallest possible measurement of a real extended material object could be called a “Quanta”. This choice of the word “Quanta” and the definition that Riemann gave it seems strange since the ideat came more than four decades before Max Planck wrote the first paper on the quantum to explain blackbody radiation, but that is exactly the way that Riemann used the word. He later addressed the very question of how his geometrical concepts were born out by experience.

> § 2. In the course of our previous inquiries, we first distinguished between the relations of extension or partition and the relations of measure, and found that with the same extensive properties, different measure-relations were conceivable; we then investigated the system of simple size-fixings by which the measure-relations of space are completely determined, and of which all propositions about them are a necessary consequence; it remains to discuss the question how, in what degree, and to what extent these assumptions are borne out by experience. In this respect there is a real distinction between mere extensive relations, and measure-relations; in so far as in the former, where the possible cases form a discrete manifoldness, the declarations of experience are indeed not quite certain, but still

> not inaccurate; while in the latter, where the possible cases form a continuous manifoldness, ***every determination from experience remains always inaccurate: be the probability ever so great that it is nearly exact***. This consideration becomes important in the extensions of these empirical determinations ***beyond the limits of observation to the infinitely great and infinitely small***; since the latter may clearly become more inaccurate beyond the limits of observation, but not the former.

Riemann noted a direct correlation between his "extension-relations" (the metrics of curvature) and the "measure-relations" of real objects. These twin concepts would affect two different areas of study of the real material world or physics, the cosmological arena of the universe at large, exactly where Einstein's theory of general relativity rules and the world of the extremely small where modern quantum theory seems to rule at present. But Riemann noted these extremes and further noted that real measurements could never be exact, thus introducing the concept of probabilities (like we see in modern quantum theory). This was exactly the problem faced by Einstein and others a half century later over the question and interpretation of the quantum theory: Was the probability in nature or in the human mathematical interpretation of nature? Riemann clearly distinguished the problem of measurement in the arena of the infinitely small where measurement might be physically impossible and thus requiring the imposition of mathematical probability over natural reality.

Riemann was quite direct in his mathematical approach to what is now called the quantum world of the infinitely small, but only indirectly defined the point/extension problem that presently plagues modern physics.

> The question of the validity of the hypotheses of geometry in the infinitely small is bound up with the question of the ground of the metric relations of space. In this last question, which we may still regard as belonging to the doctrine of space, is found the application of the remark made above; that in a discrete manifoldness, the ground of its metric relations is given in the notion of it, while in a continuous manifoldness, this ground must come from outside. Either therefore the reality which underlies space must form a discrete manifoldness, or we must seek the ground of its metric relations outside it, in binding forces which act upon it.

And in this context, where science now looks at the infinitely small 'discrete' world of the quantum, Riemann's world of a "discrete manifoldness", science needs to look for evidence of the metric curvature of the space-time continuum (as put in today's language of physics) outside of this material world where the points of space are originally bound together to form the continuum of curved space-time. In other words, the dynamical forces that bind objects together in three-dimensional space or later in four-dimensional space-time reduce to kinematical quantities or simple motions when we "seek the ground of its metric [geometric measurement] relations outside it" as extrinsic curvature in a higher-dimensional embedding space or manifold.

Einstein developed special relativity to overcome some of these inconsistencies, which unbeknownst to Einstein at the time (by his own admission) had nothing to do with the failure to detect the luminiferous aether in the Michelson-Morley experiments. This fact was demonstrated more readily and indeed forcefully when Einstein's concept of relativity was further developed by Minkowski using a four-dimensional space-time framework which simplified the physical concepts involved. Within Minkowski's framework a new line-element was used that included time and hence formed a space-time continuum that Einstein was later able to use in general relativity.

$$(d\tau)^2 = (dx)^2 + (dy)^2 + (dz)^2 - \mathbf{i}(\mathbf{cdt})^2$$

The inclusion of time into the line-element changed its meaning so that it now represented the proper time **dτ** instead of the Riemannian line-element **ds**. In fact, this equation was interpreted to represent a hyperbolic geometry because of the negative sign for the time element in the equation, at least before Einstein extended the notion to represent curvature in a Riemannian spherical geometry. Yet the concepts expressed in this relationship were thought to refer to the whole of the space-time continuum in the large as expressed by Einstein. Einstein's concept of intrinsic space-time curvature was based on a Riemannian overall universal positive (spherical) curvature that rendered the universe a finitely extended but unbounded four-dimensional space-time sphere.

Einstein's geometrical model worked so well for explaining gravity that scientists seemed to have stopped looking for and even thinking about the embedding higher manifold and just assumed that the embedding dimension was time after Minkowski developed his space-time framework for special relativity. Time may have seemed to be the n+1- dimensional manifold for the three-dimensional continuum, but that interpretation was not necessarily or overtly adopted. Few made that claim, but most just ignored the notion of a higher-dimensional embedding dimension that was required by Riemann by default. The answer is simple though. Time is not the n+1-dimensional embedding manifold that Riemann envisioned at all. That distinction would be left for the fifth space-like dimension that embeds the entire four-dimensional space-time continuum, but even then two problems remain: (1) the point-elements of which Riemann spoke were still not dealt with adequately while (2) Riemann never indicated in any way what type of geometry the embedding dimension might display.

Whether someone is talking about the line-element **ds** or the proper time **dτ**, the geometric problem of the point is still the same: Even in its approximation as ds→0, the shortest length of the line (extension) **ds** before reaching zero would still represent a line segment or line-element containing an infinite number of infinitesimal points. However, the shortest possible infinitesimal line-element cannot be built from either a finite or infinite number of points placed side-by-side to form a continuous extension. To construct a continuous line, points must first be contiguous, *i.e.* the points must touch each other with no empty space between them to be truly contiguous, but the geometrical point, being dimensionless, that touches another such point will automatically become that other point (in essence fall into it or overlap it) without forming an extension of two points.

In fact, an infinite number of points placed within a single point does not change the point in any way (placing an infinite number of nothing into nothing yields nothing, which is a weak mathematical condition) because the infinitesimal point has no dimensions to act as boundaries to limit the number of points contained in it. In his original development of differential geometry this is exactly why Riemann worded the relationship between point-elements and a continuous extension as he did. Even then, a discrete point filled with an infinite number of discrete points still forms a continuity of sorts between the infinitesimal point (discontinuity) and all line elements or segments that are approaching the point in size in that they all contain continuously an infinite number of points. But how can a true individual discrete point also be a point filled with an infinite number of like points? This indicates another level of stating the same problem.

As a strong methodical condition, we can have a true discrete individual point which must be or form the discontinuity, or a weak mathematical condition, which is the minimum necessary to form a discontinuity in the form of an infinitesimal point containing an infinite number of like points. Those theoreticians who developed non-Riemannian geometries assumed and utilized the second or weak condition but ignored the strong condition, which requires an external embedding manifold altogether. Kaluza chose to tackle the strong condition utilizing a true discrete point, which according to Klein's extension likened the Kaluza model to the discrete quantum theory, but ignored the weak condition. For their own purposes, modern quantum theorists have ignored the problems established by these two mathematical conditions altogether and seem to mix them up or change their model according to any given situation to take advantage of one or the other conditions. This practice makes modern quantum theory all the more ambiguous and interpretation prone than it already is.

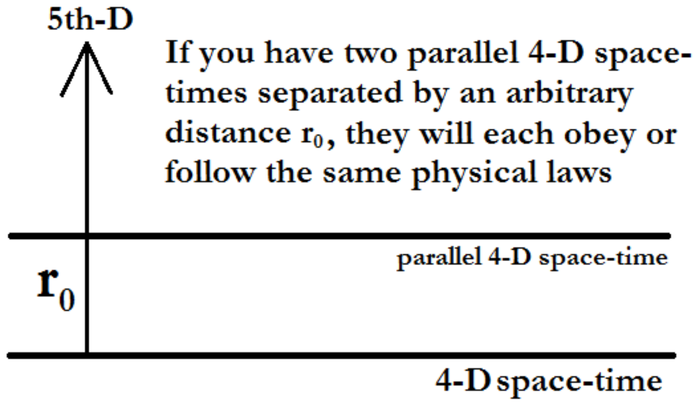
The early non-Riemannian (tangent space) geometers made use of this simple geometrical truth of an infinite number of points within the single point in a line segment to develop their own geometries of space from the availability of an infinite number of points in the tangent point of contact between the Riemannian space and their tangent spaces. But even those tangent spaces missed the point in that those spaces may have exhibited various forms of assumed affine connections with other points in three-dimensional space, but they never demonstrated or proved that neighboring tangent spaces were connected to each other either three-dimensionally outside of the three-dimensional Riemannian space or even three-dimensionally within the Riemannian curved space.

Yet Riemann hinted at that very solution to the problem when he stated that every *n*-dimensional space is embedded in *n*+1-dimensional manifold. He thus implied that the whole of the *n*-dimensional surface (space), rather than the individual points in the manifold (as Kaluza attempted), was extended as a whole into the embedding manifold or space. Beyond that he gave no indication on how to proceed from this point to describe that embedding space or manifold and thus came no closer to solving the point-extension problem. The equivalent number line problem in arithmetic or number theory can also be easily stated. The number line is assumed to have no gaps, even given the transcendental nature of roots and other transcendental numbers, but there is no mathematical proof of this assumption. Yet even this problem has a solution that is analogous to the geometrical solution of the point/extension problem.

Einstein and Bergmann: One step closer

Einstein and Bergmann (1938) actually came closer than anyone else to solving this conundrum simply by demonstrating mathematically how parallel three-spaces, all perpendicular to a continuous A-line extending from a single point in our primary three-dimensional space in the fourth direction, would have the same physics. This seems to be the first attempt of anyone, possibly since Clifford's day, to say something quantitative about the higher embedding spatial dimension. They did not address the central problem of the geometry from the point viewpoint, but still considered the relationship between an individual point and the continuity of any three-dimensional space along a perpendicular line (A-line) extended along the fourth direction in the embedding space. Yet having taken a step toward recognizing the central point/extension problem, they still did not make any statements or draw any conclusions about the overall geometrical structure of the embedding higher space or how individual points could be connected together to constitute the parallel three-dimensional spaces. They did not specify how the consecutive three-dimensional spaces or surfaces were related to the individual points that constituted them and did not account for the four-dimensional line to which they were all perpendicular. They merely assumed that the embedding dimension was macroscopically extended and closed as a whole with respect to the embedded three-dimensional space. In other words, they did not go far enough in their discovery of new geometry.

Einstein and Bergmann proved mathematically that any three-dimensional space at a distance R in the extra dimension would have the same physics as our own three-dimensional space and no more.



Einstein reasoned that individual points were closed in Kaluza's five-dimensional model, but the higher fifth dimension itself was not closed. Like Kaluza's model, the points in three-dimensional space still generated equal length lines that formed closed loops in the higher fifth space-like dimension. They did not determine how these lines came together to form the overall closed higher-dimensional space. So closure of the three-dimensional surface as a whole, which was spherical, only implied but did not necessitate closure through the higher dimension with respect to four-dimensional space-time. So they could not determine the manner of its closure (single or double polar) or what affect that closure would have on the normal three-dimensional space or four-dimensional space-time continuum.

Their structure implied that an infinite number of parallel four-dimensional space-times along the extended A-line in the fifth direction of space-time, constituting a specific 'thickness' of such elements, can exist without changing the physics of our normal experienced four-dimensional space-time. Furthermore, anything done along the line in the fourth direction of space would affect or influence the physics of four-dimensional space-time. So, all other five-dimensional models of space-time are incomplete because they are based on the simple geometrical fallacy that space can only be modeled by a simple metric or extension-based geometry ($\mathbf{F}=m\mathbf{g}$ or $\mathbf{T}_{ik}=\mathbf{G}_{ik}$) even though space really has two distinct parts – point and extension. You cannot just extend true discrete points individually into the higher dimension, without extending the whole of the continuous lower-dimensional space point-by-point into the higher dimension and close that extended model to the lower dimensional space without saying something about how the discrete points in the three-dimensional surfaces involved connected to the three-dimensional surfaces involved.

In other words, the continuity of three-dimensional space (and time) is implied but not specifically expressed in these hyperspace structures. So Kaluza's model suffers from the same point (discrete/continuity) problem as the other intrinsic-point attempts to develop a unified field theory. Simple electromagnetism already has two parts – scalar (extension) and vector (point) potential – that demonstrate how the electromagnetic field as a whole interacts with the two parts of physical space – extension and point, respectively – so gravity must act and react in the same way to the same shared three-dimensional space with two parts – scalar and vector potential – across its hyperspatial extension (five-dimensional) as well as normal three-dimensional experiential space. This is the same point that the original mathematicians and scientists working to unify general relativity and electromagnetism sensed and indirectly tried to make, but the message got lost in their attempts to express that they only sensed subconsciously and thus through the ensuing decades. So all of the geometries used in unification were **incomplete for the same reason.**

There is **one and only one solution** to this problem and it was implied in the Einstein-Bergmann model of 1938 as well as by Riemann himself in his original paper on differential geometry of surfaces. Only Clifford and those who followed his work attacked the problem directly, but ultimately unsuccessfully. The geometry of the embedding dimension must be completely specified to guarantee the continuity of discrete points in three-dimensional space with all other discrete points in three-dimensional space across the whole of the embedding dimension. Among other warnings and factors that implied the proper solution to the problem, Schrödinger's 1944 derivation of a metric (extension) geometry as a specialized case of the affine (point) geometry implied that a point geometry extension of Riemannian geometry that would yield the necessary physical characteristics of the higher embedding space or manifold should be possible. In fact, Schrödinger's derivation naturally yielded an extra factor that corresponded to Einstein's earlier cosmological constant and today's Λ-CDM, which is usually needed to explain dark matter. This demonstrates that dark matter and dark energy are a secondary effect of normal gravitation as well as the simple fact that dark matter is a product of normal baryonic matter and not a separately constituted point particle as assumed within the standard model of the quantum.

Clifford certainly thought that the physical and geometrical characteristics of individual discrete points in three-dimensional space were essential for understanding Maxwell's electromagnetism, so he tried to develop his point geometry of the fourth dimension of space based on twists and bi-quaternions. He had already established the basic idea that normal matter was nothing but curvature (hills and such) and motion was nothing but ripples moving through the curvature in a Riemannian embedding manifold. Since it was well known that any object moving into or through the higher dimension returned to three-dimensional space after exchanging right orientation for left (Newcomb, 1870s, Wells 1890s), the higher dimension that Clifford and others were investigating was obviously single polar Riemannian. Yet it seems that no other researcher has ever tried to further develop the Riemannian concept of the embedding dimension and rather sought to develop other geometries within the points (such as the later non-Riemannian geometries) and retain the intrinsic nature of Einstein's space-time continuum. For theoreticians before Einstein, the curvature of space was real and extrinsic although they would b]not have used the term extrinsic since they did not even think in terms of intrinsic until general relativity was developed and philosophical positivism forced the issue.

IV. The only possible geometrical solution

Ascension of points

There is obviously and undoubtedly a fundamental geometric disconnect between the concept of discrete points and the extended surfaces constructed from those same discrete points. This conceptual void has been filled throughout the past with infinities and infinitesimals, neither of which is clearly or concisely defined with regard to an absolute number of points, *i.e.* they are not visualizable and thus mark the limits where human reason breaks down. While both the differential geometry of surfaces and calculus use diminishing (either the distance along an extension or the duration of time goes toward zero) extensions to visualize the concept of a point and explain space itself, those extensions always consists of an infinite number of points until they are thought to go to zero at a single discrete and dimensionless geometrical point. Reason tells us that the infinite sequences of mathematical limits must end at some point or they would not be discontinuities and thus the reality of discrete (truly unique and individual) points is never any more than a logical implication.

So it seems that the opposite argument that a mathematician should be able to theoretically and mentally build an extension out of such individual discrete points, even if such a construction is physically impossible, should be true, but that conjecture is not. Any attempt to connect points to make an extension runs into a very serious problem: There exists no method or logical argument by which dimensionless discrete points in space contiguously (while touching at their outer edges) could be placed next to each other to form a continuous extension let alone the extension-based geometry necessary to represent the concept of space in physics. So, even a mathematical mental construction of extended lines (simplest one-dimensional surface) is impossible.

Yet an infinite number of points can be dropped into a single discrete point without changing that single discrete point, by either its inherent value of properties, which has allowed the theoretical development of completely discrete geometries (non-Riemannian and tangent spaces) within the discrete points that are thought to constitute normal space. The high accuracy of modern quantum theory in the form of Feynman diagrams actually depends on this characteristic of points while the only method that even comes close to forming an extended space from ideal individual mathematical points is the quantum method of perturbation. Perturbation mathematically smears a dimensionless point out over an already existing three-dimensional space so integration and other mathematical methods that depend on continuity can be performed on the individual discrete points. This strictly quantum method gives rise to such misconceptions as a fuzzy point, quantum foams and quantum fluctuations to mimic continuity at the lowest possible imaginable level of physical reality.

It thus seems that a mathematical method of generating a space or spaces of as many dimensions as deemed necessary from dimensionless points should have been a previously well developed concept since the inverse logical argument is a necessary requirement for mathematical rigor in both geometry and calculus. But this method has never been developed and probably never even questioned by mathematicians. This oversight creates a gaping hole in mathematical logic, to put it lightly, especially in cases where mathematical logic applies to physical realities such as our commonly experienced three-dimensional space. There is hope, however, for a solution to the problem since any length of a line or size of a surface diminished all the way down to zero as well as the zero point consists of an infinite number of points, which forms a continuity of sorts on its own merits. Furthermore, if such a point is filled with an infinite number of like points, it cannot be considered truly discrete anymore and only a truly discrete singular dimensionless point can alone and uniquely disrupt either a geometrical or physical continuous surface to form a discontinuity.

This discrepancy in the conceptual foundations of mathematics has never been discussed because of the inherent problems associated with the different and mutually incompatible concepts of a point. For example, to form a continuous extension, two points must at least be contiguous, *i.e.*, making contact or touching, but neighboring dimensionless points cannot make contact with each other without becoming each other since they are dimensionless. To solve this fundamental conceptual problem, it would be necessary to derive and establish the legitimacy of a mathematical theorem that guarantees the reality of a point-generated physical space. Such a theorem-based mathematical system would depend upon the ability to generate extensions as well as real spaces from the ideal dimensionless points that constitute the space, *i.e.*, the dimensions of space must be shown to depend upon the dimensionless points in that space. So this theorem could be described as a 'reality' or 'existence' theorem in mathematics since it applies to the reality of a physical space: It establishes the mathematical prerequisites and characteristics necessary for our commonly experienced physical space to exist. The whole question is also directly related to the concept of a physical field or fields that fill real space. Michael faraday is perhaps the only person to ever discuss this important issue and he discussed it within a purely physical context in his original paper on the field concept in 1840.

The major obstacle to solving this problem would be how to define contiguity of the individual dimensionless points in a way that would allow the construction of the continuous extension. Although this may seem impossible, a working definition of contiguous dimensionless points must be established first. Two discrete dimensionless points, A and B, could never be contiguous by contact of their outermost boundaries because, having no extension or outer boundaries and being dimensionless, any contact between contiguous points would render them 'overlapping' or coincident. However, two different independent dimensionless points could be considered contiguous without actually depending on contact between them if and only if they were so close that no other dimensionless point could be placed between them to separate them.

This situation is hard to imagine, but the concept is mathematically valid. If 'nothing' or 'no-thing' is between them to separate them by any measurable distance, which for a point is 'nothing', they could only be contiguous by default, but still contiguous enough to determine a minimal extension. Given this concept, two dimensionless points, A and B, in close proximity to each other could be considered contiguous for all mathematical intents and purposes if they were so close that another dimensionless discrete point could not fit between them to separate them. If "no-thing" as well as nothing is placed between them to separate them by any distance, even when that distance is nothing, they must be defined as contiguous by default rather than merging into a single point. Their contiguity defines the first beginning of a single dimension which is a prerequisite for a real measurable extension.

Now that the points are legitimately contiguous, other dimensionless discrete points can be placed in positions contiguous to them to construct at least a one-dimensional extension of measurable length. This cannot be accomplished within the space (in this case a one-dimensional space of line) they are generating. There are literally an infinite (indefinable) number of directions either point could go to position itself contiguous to the other point. However, this problem can also be overcome. The points are only restricted to be dimensionless in the dimension(s) of space they share, which would constitute real physical one-, two- or three-dimensional spaces. So what then would guarantee the reality of a three-dimensional space given this situation and conditions? To 'prove' the validity of any mathematical system it is necessary to go outside of that system since only the consistency of a system can be 'proven' from within that system.

A mathematical 'theorem' that states this exact idea was first developed by Gödel and was named after him, however this exact same idea was suggested for physics nearly a century earlier by Riemann himself.

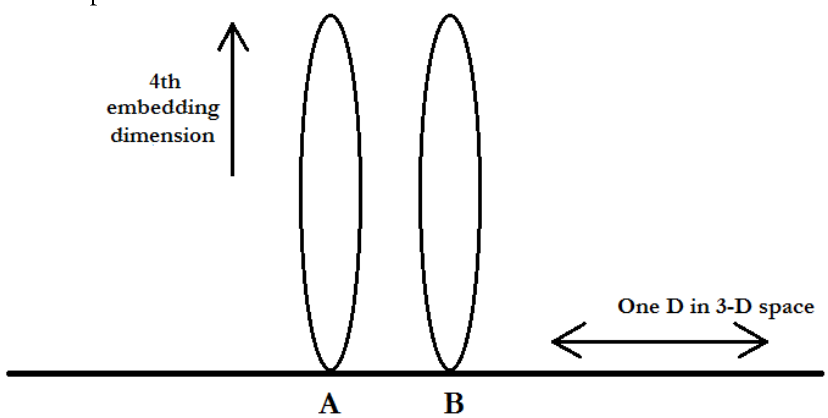
> … in a discrete manifoldness, the ground of its metric relations is given in the notion of it, while in a continuous manifoldness, this ground must come from outside. Either therefore the reality which underlies space must form a discrete manifoldness, or ***we must seek the ground of its metric relations outside it***, in binding forces which act upon it.

According to Gödel's theorem, only the internal logical consistency of a mathematical system can be proven within that system and in the words of Riemann, the internal continuity of a manifold or space "must come from outside." So the validity (reality) of a mathematical or geometrical system, based on the primary foundational theorem from which the system is generated, can only be determined or proven logically true from outside of the system. All that present mathematics or physics can determine – prove or verify in either case – about our commonly experienced physical space is the logical consistency or rationality of the geometrical system that best characterizes it based upon the geometrical theorems and/or physical theories of the three-dimensionality of space. A reality theorem in mathematics could only depend on dimensionless points in three-dimensional space utilizing a higher four-dimensional embedding space to guarantee that the three-dimensioned space could be generated from individual discrete dimensionless points.

This solution to the problem is already implied in Bernhard Riemann's original concept of space curvature whereby an *n*-dimensional space is embedded in an *n+1*-dimensional manifold, but it seems to never have been undertaken. So the only way to mathematically 'prove' the existence of our commonly experienced physical space is to go outside of (or beyond) it and the only way to accomplish that is to study the higher or *n+1*-dimensional manifold in which it is embedded. Science needs to determine what kind of an *n*+1-dimensional would yield our three-dimensional space with its own specific set of physical properties before physics can proceed any further than it already has because physics has now foundered upon the shoreline of the boundary between point and extension. Since science knows nothing, nor could science do anything more than speculate about possible properties of space external to our three-dimensional material reality, the starting point for determining the physical properties and characteristics of the higher-dimensional embedding manifold could only come from inference derived from the most fundamental physical properties of our common three-dimensional space and work backward to determine the properties of the n+1-dimensional embedding manifold. Fortunately, those exact fundamental properties are just the items so far determined and discussed: point and extension, discrete and continuous.

Einstein and others have already interpreted, or better yet misinterpreted, the notion of space-time curvature as an internally closed three-dimensional space coupled to a fourth dimension of time, as is the case if curvature is intrinsic to the space-time continuum. This interpretation does not 'necessarily' represent the true physical situation, but rather a Machian positivistic interpretation of one possibility. There are no changes in the mathematics or the physics if the curvature in Einstein's theory is interpreted as extrinsic within a higher embedding dimension. In other words, they either did not take Riemann's statement that seriously or they did not understand what he meant by it. Yet their work can be of help, even though it is not complete enough in itself to solve the overall problem. In Einstein and Bergmann, a macroscopic extension was implied and closure of the higher dimension as a whole across its extension (instead of point-by-point) was analyzed, but continuity was not 'proven' between any individual points in the three-dimensional extension emanating from a line extending in the fourth direction or between individual discrete points and other such points in the three-dimensional extensions. A first step was made though by Einstein's proving that the physics in every parallel three-dimensional space displaced along the line in the fourth direction of space would be the same, which is also important. It is only necessary now to prove that discrete points can be used to construct the continuity of three-dimensional space.

In the next step of the geometric 'proof', perpendicular lines can be drawn in the fourth direction of space from two points, A and B, such that they are an infinitesimal distance apart in three-dimensional space. The points A and B are contiguous because they are so close to each other that no other dimensionless point can be placed between them.

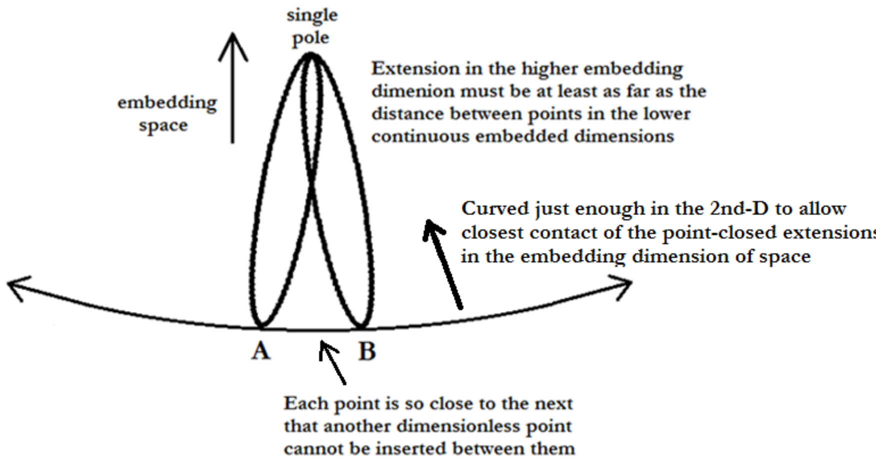


These lines would normally remain parallel in the fourth direction and equidistant apart no matter how far they are extended, which is a direct violation of the very concept of the non-Euclidean geometries that were developed to take into account the possibilities presented by the failed parallel postulate and which does nothing to verify the possible physical reality of the three-dimensional space.

Strangely enough, this particular geometrical structure corresponds exactly to the point-particle case expressed in the standard model of quantum physics which says nothing about the geometry of physical space but does seem to require a flat space. Each point is completely discrete within itself and has no distinguishable physical properties, not even mass, yet the continuity of the quantum field connected to that point-particle by bosons is assumed. The bosons, which are point particles themselves, carry physical properties of the field and are exchanged with point-particles such as protons to give them the physical properties by which they are measured. The boson fields seem to be no more than a specialized from of Newtonian absolute space that is defined only by the points which make it up without any inherent extensions and thus Euclidean flat by default.

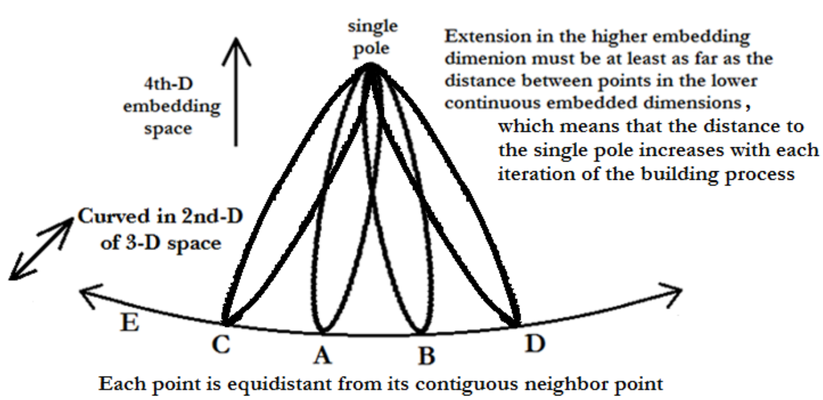
So this model does not and, in fact, cannot answer the important question of how the boson fields can be continuous (the same is true for any quantum field) or how they can carry physical properties when they are also point particles? Quantum theorists just ignore this problem, just as they ignore the concepts of geometry and the real extension of particles. And like this geometrical situation, there is no way to mathematically or physically verify the physical properties of the quantum fields. The same situation exists for discrete points in Euclidean flat space, Newtonian absolute space, the intrinsic curvature (which is a contradiction in terms) of standard general relativity, quantum theory, the standard model in quantum theory, any point-particle model in physics, quantum loops, Kaluza theory, Kaluza-Klein and all superstring and related theories. This problem is ubiquitous such that all of physics and mathematics suffers from this same problem in one form or another, but it is especially crucial for the standard model of the quantum, which is basically a non-geometrical theory that defaults to a Euclidean flat pseudo-geometrical structure for space-time.

However, if the one-dimensional line in three-dimensional space is curved in a second dimension, then the lines drawn from the points will draw closer three-dimensionally the further they are extended in the fourth direction until they meet at a single point before the lines return to the same point from which the lines originated. Points A and B have now formed individual closed loops like Kaluza's A-lines, however, unlike Kaluza's A-lines the two lines form a single closed embedding manifold as suggested by Einstein and Bergmann.



Once they have moved at least as far as the infinitesimal distance between them the A-lines would meet. So a continuous extension can only be mathematically proven for a one-dimensional line curved a second dimension that yields a Riemannian curved space of two dimensions.

Now take another point, C, using the same restrictions for contiguity as for A and B, but in the opposite direction in three-dimensional space and repeat the procedure. C and A will coincide at one point in the embedding direction that is at least equal to or greater than the infinitesimal distance between them in three-dimensional space. In fact, A, B and C will all coincide at a single point in the fourth direction. Add two more points to either side of B and C – D and E – following the same procedure these points will also coincide at the same point in the fourth direction as A, B and C.



Eventually an infinite number of points to either side of B and C would converge and form a closed space in one of the three-dimensions of three-dimensional space. All of these points would coincide at the same point in the higher fourth dimension of space and all loops (A, B, C and D) would be of equal length. This structure, as it is now set up, already implies that non-material fields would follow an inverse square law as they spread out in the curved embedded space and light waves traveling in one direction would spread out spherically in the other two dimensions of space. The inverse square law is one of the fundamental characteristics of our common three-dimensional space.

The greater the number of discrete points that are included the further away the single pole point moves from the two-dimensional construction until the curved line in two-dimensional space closes on itself to form a closed circle and end the construction process. This process forms a unique embedding space or manifold and yields a double-polar Riemannian three-dimensional embedded space with the same physical characteristics – *i.e.* symmetries, translational and rotational motions – as our commonly experienced three-dimensional material space embedded in a single-polar Riemannian manifold. Individual points in three-dimensional space can now remain discrete within three-dimensional space yet still be continuous with all other points in three-dimensional space via their connection in the higher embedding dimension.

When the same procedure is conducted for the other two dimensions of three-dimensional space, all extensions in the fourth direction of space would coincide at a single point that is at least as far from three-dimensional space in the fourth direction as the sum of an infinite number of infinitesimal distances that separate the infinite number of points that make up the closed space in any one dimension of the three-dimensional space. The real three-dimensional space that is formed by this logical procedure would be internally double polar elliptical (spherical), but externally single polar elliptical in the fourth direction. The embedding higher-dimensional space is single polar elliptical (spherical) and at least as large as all of the dimensions in the real closed three-dimensional space. This space is exactly that type or form of physical hyperspace proposed by William Kingdon Clifford and envisioned by other mathematicians in the late nineteenth century after they were first introduced to Riemann's geometry.

This structure, however, does not yet completely answer the question of how the individual dimensionless points in three-dimensional space can be contiguous to each other and thus form continuity in any given direction within three-dimensional space. The answer is implied by the geometrical structure and can be easily explained by reversing this process. If instead we start from the single polar point in the fourth dimension where all points in the three-dimensional subspace coincide and move away from that point along the fourth direction toward the full three-dimensional spherical space the order by which the individual dimensionless points of three-dimensional space separate would determine their order and placement in the three-dimensional space. Therefore the reality and existence of three-dimensional space built of dimensionless points is guaranteed. The interesting part of this procedure and the ensuing geometrical structure comes in the physical attributes that emerge for our own physical space of experience.

Physical properties of the embedding manifold

The mathematical conditions placed on this higher physical space that are required to render three-dimensional space real are quite straightforward. They are four in number. The first two are easily recognizable: (1) a one-dimensional line extending in the fourth direction from a dimensionless point in three-dimensional space must complete a circuit and return to that same point from the opposite direction;

and (2) all lines extending from all dimensionless points in three-dimensional space must be of equal length. These are just the mathematical conditions that Kaluza set on his five-dimensional extension of Einstein's four-dimensional space-time continuum. The third and fourth conditions are not so easily recognized: (3) the one-dimensional lines extending into the higher dimension from three-dimensional space must all be at least as long as a circumference line that completely encircles the embedded three-dimensional space. That is because (4) all of the one-dimensional lines that extend from the dimensionless points in three-dimensional space must pass through a single common point in their circuit before returning to the three-dimensional space in the opposite direction.

These last two conditions can only be fulfilled if the higher embedding space is macroscopically sized, just as Einstein and his colleagues determined or assumed in the 1930s. It must also assume a single-polar elliptical Riemannian shape. In laymen's terms, each of the one-dimensional lines extended in the fourth direction of space works like a Möbius strip. This means that each point in three-dimensional space would have an inherent half-twist to it. These last two conditions impose the same overall structure as the four-dimensional space that Clifford envisioned in the 1870s, but the physical consequences of these conditions go beyond what Clifford imagined. They also rule out the possibility that the superstring theories that depend on infinitesimally small and compacted higher dimensions could possibly represent physical reality.

The half-twist in each dimensionless geometrical point of our real three-dimensional space means that rotations of extended lines around a central point are possible in our common experiential space. In other words, three-dimensional space can be characterized by its support of either translational or rotational motions, which just happens to be observationally and experimentally true. Furthermore, this twist to the points in three-dimensional space accounts for the half-spin of elementary material particles and, in fact, establishes a geometrical requirement that all real stable material particles have half-spin. So the bundle of A-lines extending into the embedding dimension and through the single pole to return to their starting point on the other side of three-dimensional space and constitutes a real material particle and every real material particle must have a net half-spin to be real and exist in three-dimensional space. Thus material particles can only be stable and real if they meet the geometrical condition of a half-spin in the higher fourth dimension. Only protons, neutrons, electrons (muons and tauons are high energy electrons in n=2 and n=3 quantum states) and neutrinos are real material particles, meaning, of course, that all of the geometrical points within the space they occupy are constrained by the half-spin of the same extended fourth dimension of space as opposed to other possible artificially constructed physical configurations.

Einstein adopted the geometrical structure of a double polar Riemannian sphere to model gravity as four-dimensional space-time curvature, but it is really space alone that is curved in the higher dimension. However, Einstein's positivistic leanings got the better of him when he interpreted the curvature as an intrinsic property of the four-dimensional space-time continuum. On the other hand, the Riemannian geometry that he used was only a metrical or extension-geometry and could not directly account for the individual points in space or the physics that depends on the geometry of the individual dimensionless points. Classical electromagnetic theory implies both an extension- and a point-geometry because electricity has a three-dimensional scalar potential structure and magnetism has a four-dimensional vector potential structure. Both are associated with points in three-dimensional space (**E** and **B**), but the magnetic vector potential (**A**) must also have magnitude and direction even though it exists within a dimensionless three-dimensional point, which implies a higher dimensional embedding space. (Beichler, 2007) Common

Newtonian gravity only implies the need for an extension-geometry, so Einstein was unable to completely unify gravity and electromagnetism within the Riemannian space-time framework let alone incorporate the quantum into his ultimate interpretation of the world geometry.

This, however, does not mean that the task is impossible. Einstein was on the right track when he adopted Kaluza's five-dimensional space-time framework in the late 1930s as well as when he adopted Cartan geometry in 1929 and the symmetric/non-symmetric tensor calculus after 1945. All of these geometric systems offer some limited form of a combined point- and extension-geometric structure. However, Einstein failed to realize two things while pursuing these particular solutions to unification. The combination of these two geometries through a Kaluza five-dimensional structure could lead to a single field theory, but Einstein only attempted his unifications of physics with these geometric structures individually. And secondly, he did not realize that the quantum could never emerge from an over-restriction of the mathematics of his geometrical structure as he hoped, but rather that the quantum is a fundamental property (the binding constant) of space-time itself.

In summary, there are five conditions.

Kaluza (relaxed cylindrical) conditions

(1) All A-lines are closed with respect to their origin points
(2) All A-lines must be of equal length

The Clifford conditions

(3) The one-dimensional A-lines must all be at least as long as a circumference line that completely encircles the embedded three-dimensional space
(4) All A-lines must pass through or come together at a single common point – This is the pole of a single-polar Riemannian sphere

The Einstein condition

(5) Any theory must explain why we do not observe the higher dimension(s)

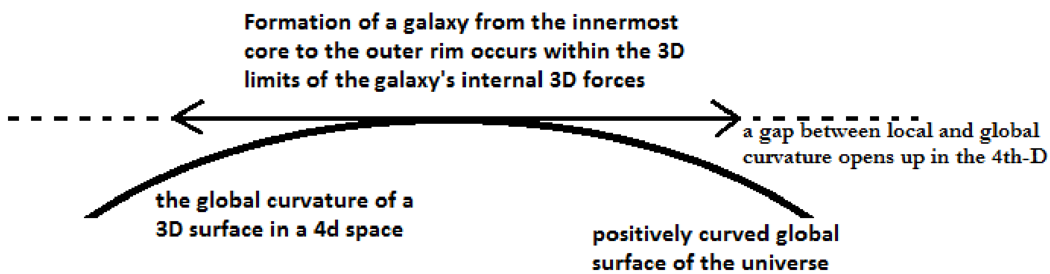
V. Geometrical consequences of a higher-dimensional structure

Practical applications to gravity

Once the higher-dimensional embedding geometry is applied, the physical consequences of adopting the higher space structure become clearer. The geometry shows how to interpret the equations, not the other way around. The best example of the geometrical interpretation is in the concepts of dark matter and dark energy. While the non-symmetric (point-geometry) portion of the Einstein gravity tensor yields dark matter and dark energy instead of electromagnetism as Einstein and others originally thought, dark matter and dark energy cannot be logically understood or even visualized unless a four-dimensional space is used. Through its analogy to electromagnetism with which gravitation is now symmetric, *i.e.* it has a one to one correspondence of properties relative to the geometry of dualistic space, dark matter could be explained as a non-local curvature of the space-time continuum the emerges as an interaction between individual material bodies and the rest of the material bodies in the universe as represented by the overall positive curvature of the space-time continuum. Dark energy can then be defined as the point-by-point gravitational or gravnetic vector potential within the overall positive curvature of the space-time continuum. Neither dark matter nor dark energy is dark. What is mislabeled as dark matter is just a non-local gravitational effect of normal matter and dark energy is potential rather than energy, so a new class of quantum particles to represent either is unnecessary.

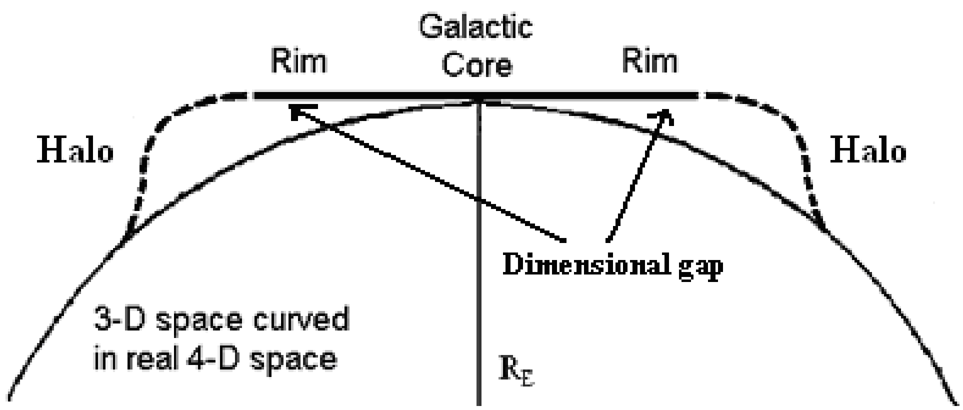
Given this new interpretation of gravity acting through the dualism of physical space, a geometrical picture of the dark matter halo of a galaxy can be drawn. As gas clouds collapse to form stars and star systems which in turn collapse to form galaxies, the extra-dimensional geometry of galaxies can be mapped out. As each star system is added to the galaxy and orbits the galactic core, it has a component of gravitationally derived velocity as well as an extra component that corresponds to the four-dimensionality of the galactic plane. The more stars added to the galactic plane, the greater the four-dimensionality and variation from the average curvature of the three-dimensional universe as a whole.

The three-dimensionality of the forming galaxy is determined by the internal three-dimensional forces of the galaxy within its own material extent and quite independent from the much weaker and thus irrelevant three-dimensional forces of the universe as a whole represented by the global curvature Γ of the universe as a whole.



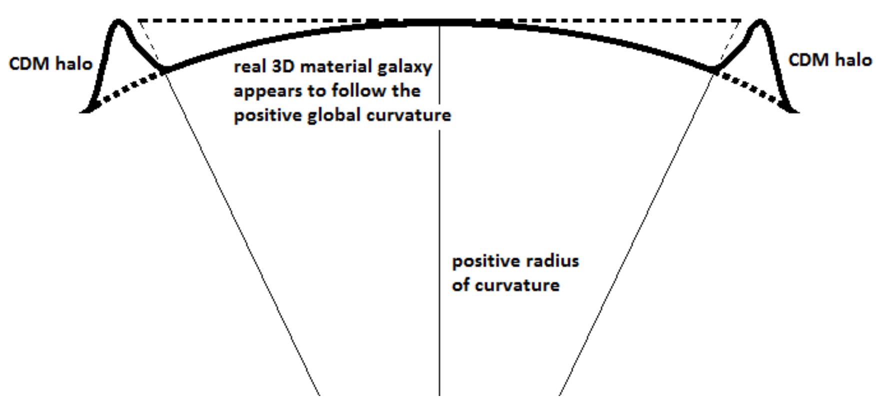
The internal three-dimensional forces of the galaxy are too great to succumb to the external three-dimensional forces of the universe, characterized by a global surface that is so very gently (weakly) curved (extrinsically) in the fourth dimension of space. The galaxy thus grows in a direction perpendicular to the radius of curvature of the universe (tangent to the curved surface at the center point of the galaxy) because the universe cannot hold it down to the overall three-dimensionality represented by the global curvature. A 'dimensional gap' opens in the fourth dimension between the three-dimensional galactic plane and the three-dimensional global surface of curvature as the galaxy expands outward. The height of this gap at any point is a function of the galactic radius. The dimensional gap is essentially the same as the passive inertia that adds to global inertia as explained above, but it is also a form of internally packaged or inertially restricted DE.

In other words, the more stars and mass added to the galactic plane the greater the galactic radius, the greater the three-dimensional galactic plane extends into the fourth dimension (tangentially to the spherical curvature of the universe as a whole) and the greater the variation of the galactic plane from the average or global curvature of the universe. DE is the gravitational vector potential gained by the galaxy's extension into the higher dimension and 'above' the global curvature.



Extension into the higher dimension is potential and when that potential interacts with matter it is reformed as DE that adds speed to the stars in the form of an extra gravitational potential energy resulting in kinetic energy.

So in essence, or perhaps 'quintessence', when the matter in the galactic rim extends into the fourth dimension it is gaining DE in the form of inertial momentum (mv) without the corresponding gain in mass from the universe as a whole as represented by the global curvature. DE, or the gravitational vector potential, exists everywhere in empty space, but when it is bound or restricted inside a particle it becomes or rather 'is' active inertia. However, when the DE is constrained or bound into a material particle by spatial curvature (under quantum conditions) it becomes or 'is' passive inertia. Within one context, DE supplies the extra speed to the stars and systems in the galactic rim, but within another it is extra inertia because it is constrained within the body of the galaxy (local curvature relative to global curvature) by the matter in the stars.



The halo forms around the galaxy to maintain three-dimensional integrity of the whole and continuity with the rest of the universe. But only local variations from the spherical universe are allowed, thus we 'observe' galaxies as part of our three-dimensional surface with the added 'halos'.

It takes dark energy to 'pull' the galactic plane down to match the overall positive curvature of the universe and absorbing that amount of dark energy (while the galaxy is undergoing its original formation) increases the orbital speeds of the star systems as the spiral arms form. The further out from the core source of gravity, the greater the 'dimensional gap' would be under the influence of normal gravitational forces and the greater the amount of dark energy absorbed from the surrounding universe. Yet the Λ-CDM halo area or ring just beyond the edge of a galaxy has no material content to pull down the curvature by absorbing dark energy and is left around the galaxy as a semi-permanent artifact of non-local curvature due to the galactic core's excessive mass, a bath-tub ring or shadow of the galactic core. In either case, the dark energy content that is added to the stars and systems in the rim as potential is due completely to the 'dimensional gap' that opens up between the local and global curvature.

Perhaps this can be more easily visualized by looking at the new Newtonian equation for the interaction of the material universe with a particular piece of local matter. The new gravity equation

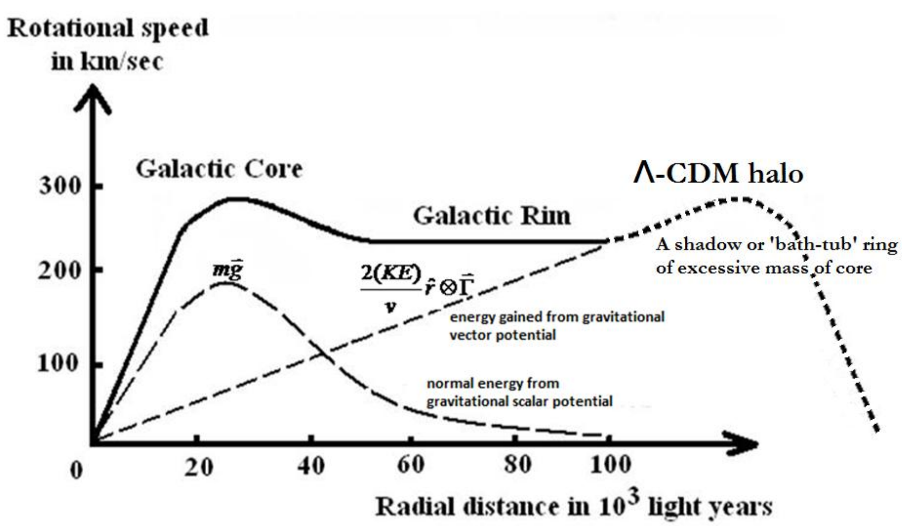
$$\vec{F} = m\vec{g} + m\vec{v} \otimes \vec{\Gamma}$$

can be rewritten in terms of simple kinetic energy as

$$\vec{F} = m\vec{g} + \frac{2(KE)}{v}\hat{r} \otimes \vec{\Gamma} \, .$$

It should be quite clear from this equation that the contribution of the global curvature to the total force in the form of the kinetic energy (applied dark energy) increases as a function of the radius and the slowing of gravitationally derived speed from the central body or local curvature (gravitational field). As each star or star system is added to the galaxy and orbits the galactic core, it has a component of gravitationally derived velocity as well as an extra component that corresponds to the four-dimensionality of the galactic plane.

This dependence is exactly what is observed as shown in the graph below.



The halo is like a shadow or mirror image of the galactic core due to the centrifugal force (equal and opposite) of the gravity of the rest of the universe represented by Γ

The curves representing the scalar and vector potential contributions to energy add to give the speed curve for the Andromeda galaxy, as shown. The kinetic energy gained from the vector potential graphs as a straight line running from the zero point (galactic center) to the edge of the galactic rim, supplying exactly the kinetic energy needed to explain the constant speeds of stars and star systems around the galactic core. Nearer the galactic core, the vector potential correction to speed is masked or dominated by the stronger scalar potential of normal gravity to give the characteristic curve of spiral galaxies in general. So when the galactic disk is 'pulled down' to conform to the overall 'spherical universal standard' curvature an amount of dark energy equal to the dimensional gap goes into the star systems in the disk and speeds them up to the observed values. However, there is no substantial amount of matter past the outer rim of the galaxy for which the extra curvature (an extrinsically induced continuity) can be compensated for by excess motion, so the excess curvature remains as a real physical artifact in the form of a curved 'bump' or 'halo' around the galaxy.

In other words, even though the galactic plane has a four-dimensional component, that component is compensated for by the excess or above normal gravitational speed because the normal matter in the galaxy is constrained to the three-dimensional surface of the universe that is common to everything material in the universe. Real matter is restricted to three-dimensionality and thus the three-dimensional surface that is extrinsically curved in the higher dimension because real matter is defined by its curvature in that surface, so the galactic plane is commonly observed as part of the global curvature and continuous with the global curvature. The halo could otherwise be considered curvature in the surface that has no local source of matter, but forms from an interaction between local (the galaxy as a unit) and global matter (represented by $\boldsymbol{\Gamma}$).

This model thus explains the physical origin of Λ-CDM and dark energy, but even more physics can be gleaned from the above mathematical model. There is still a matter of HDM or hot dark matter that permeates the universe as a whole. While dark energy permeates the whole universe as point-by-point vector potential, which more-or-less accounts for the 'thickness' of three-dimensional space within the extra embedding dimension, the metric curvature from point-to-point throughout the whole of the spherical structure of the universe has not been identified. However, that small amount of local curvature can be easily associated with hot dark matter. It is not specifically particulate unless it would be normalized to the specific quantum and geometrical conditions that define material particles. However, the smallest amount of curvature possible that could be likened to a measurable (physically interacting) particle would be the neutrino and CDM has been described as possibly consisting of high speed neutrinos.

The source of the basic spinning motion of galaxies has also long been a matter of mystery. Some scientists have even suggested that the universe as a whole is spinning which would subject galaxies and other large 'objects' to a three-dimensional Coriolis-like effect and thus explain their spiral structures. But this mystery is more easily understood within this new model of gravity. In the centripetal force experiment, the spinning or rotation of the apparatus caused the centripetal action, but in the case of galactic formation the pre-existence of the cross product indicating the secondary effect of twist or spin inherent in every point in the gravitational field causes the galaxy to rotate around a common center point as matter is attracted inward during the accretion phase of galactic growth and formation. The spin which is characteristic of material bodies ranging from the ±½ spin of elementary particles to whole galaxies exists in each inertial physical point of the world geometry. So not only does the halo originate from the new gravity term to keep stars in the galactic arm moving at constant speed, independent of their distance from the galactic center, but it also creates the 'spin' of the galaxy itself around the galactic center or axis of rotation.

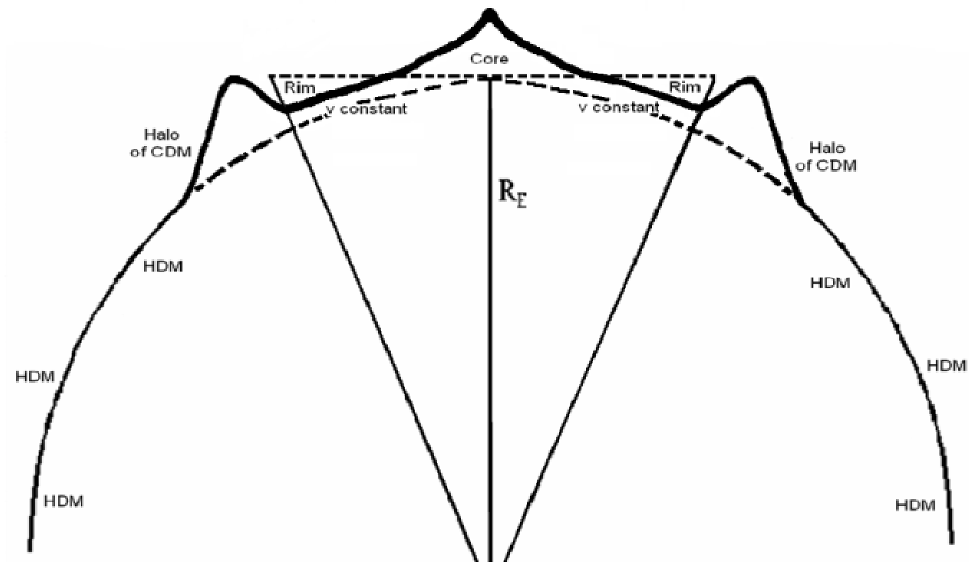
Predictions

Various astronomers during the late nineteenth century actually looked for spatial curvature since they first interpreted the new non-Euclidean and Riemannian geometries as physical possibilities rather than mental mathematical abstractions. Having found nothing in their parallax observations that would indicate spatial curvature, they assumed that if space were curved and non-Euclidean the curvature would be so small, approximating Euclidean flat, that it fell within the error of their observations. However, had they known of the existence of spiral galaxies at that time and come to the conclusion or observation of what cosmologists now call Λ-CDM, there can be little doubt that they would have interpreted the galactic halos as evidence of the non-Euclidean nature of curved space. It is only today's scientific biases and prejudices that requires the interpretation of dark matter and dark energy as particulate in nature rather than geometrical and structural properties of space or space-time when all evidence points to the latter explanation.

Otherwise, had astronomers paid attention to Heaviside's equations in the 1890s, they could have even might have predicted the existence of an unspecified and normally unsuspected 'halo' of spatial curvature around the observed three-dimensional galaxy even though that 'halo' curvature is not directly related to the gravitational curvature set by the core's matter. From our relative position within the three-dimensional material surface, astronomers could only view the halo and the galaxy as they appear within the same three-dimensional surface that provides our perceived material reality, even though the galaxy is actually extended beyond the global curvature of the surface that is the three-dimensional universe. In other words, galaxies larger than a certain minimum radius, as defined by the material content in their cores, could only exist in three-dimensions if the halos exist as described. Otherwise, those galaxies would extend out into the four-dimensional space at large and they would be invisible to astronomers and observers, except for their central cores.

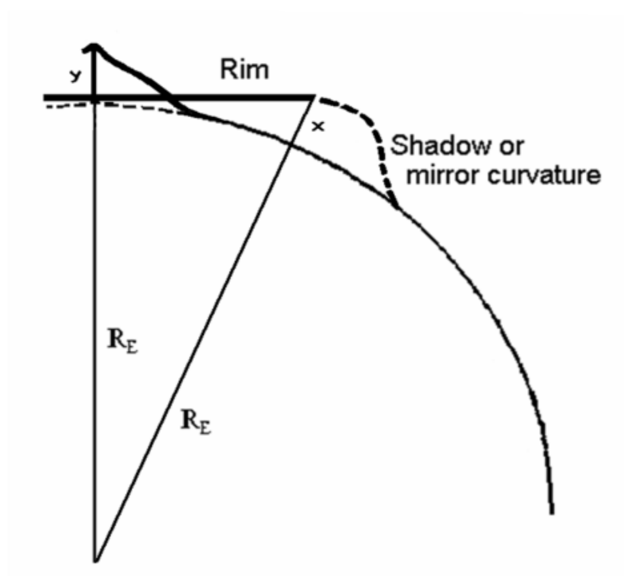
In reality, the halo evolves a small amount at a time as the core and spiral arms of the galaxy emerge from primordial gaseous clouds. As the core forms and matter accretes to the core over time to form a galaxy, from the center outward, the gravitational forces of the clumping matter act in only three dimensions. So the galaxy forms outwardly along a three-dimensional tangent line that is perpendicular to the radius of the four-dimensional positively curved surface of the universe. According to simple relativity theory, matter curves space-time rather than the reverse, so the very slight overall global curvature of space-time is not strong enough to 'pull' the accreting galactic plane of matter 'down' to the surface of the positively curved universe as the galaxy forms. As the material galaxy grows by the accumulation or accretion of material bodies, the halo also grows in direct proportion to the radius of the galactic body, until the building (or material accumulation) phase of the galaxy is complete and the radius stabilizes at a roughly constant value.

A far more accurate view of the phenomenon would include the local curvature due to the matter within the galactic core. The core matter provides the gravitational forces that direct and guide the various star systems in their orbits around the galactic axis. The orbiting bodies actually follow geodesics along the 'extrinsically' curved surface of four-dimensional space-time due to the presence of matter at the core, but appear as three-dimensional elliptically curved paths within the three-dimensional surface of the Riemannian surface. In this case, the curvature would represent the gravitational field potential that orbiting star systems and other material bodies follow.

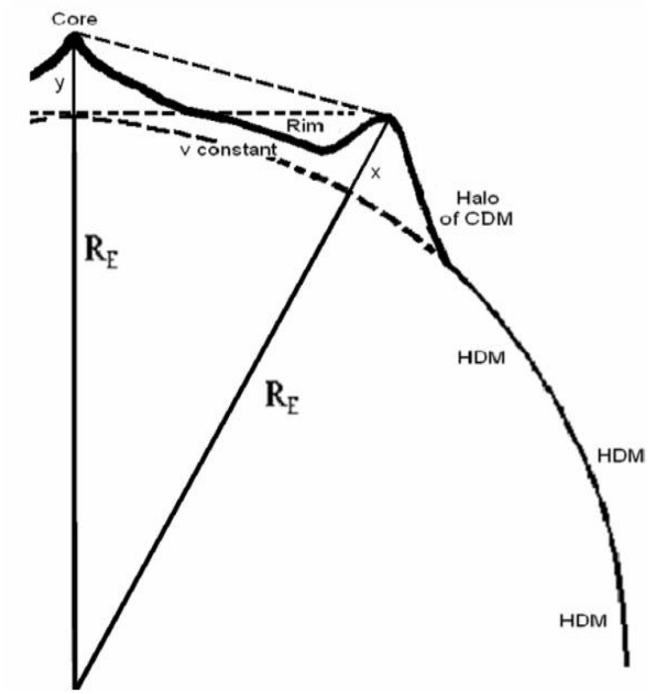


The rim of the galaxy, which conforms to the overall global curvature of the space-time continuum between the galactic core and the halo, would form a gravitational equipotential surface. In other words, the speeds of all objects lying in the galactic rim would move at approximately equal speeds, except for local variations, if they travel along the equipotential surface.

The simplicity of the four-dimensional explanation of CDM is not just limited to the physical model. Many calculations dealing with this new view of curvature can be reduced to a matter of simple geometry, simplifying the mathematics involved. For example, the Pythagorean theorem can be used to determine the extent or amount of curvature, and thus the amount of CDM, at any point along the galactic plane.



The Pythagorean Theorem can be used to calculate the degree and extent of the non-local curvature that constitutes the halo. In this instance, the amount of curvature surrounding the Andromeda galaxy is easy to calculate. According to the Pythagorean Theorem, we have



Given the calculated value of the halo curvature, the amount of curvature at the center of the galaxy could then be found by simple triangulation.

$$R_E^{\,2} + R_{Andromeda}^{\,2} = (R_E + x)^2 ,$$

where x is the extent of curvature at the boundary between the galaxy and the halo. Using values of 10^{10} light years (Butterworth, 2005; Blair, 1996; Cornish, et.al., 2003) for the Einstein Radius and 10^5 light years for the radius of Andromeda, the extent of the curvature would be about 0.5 light years or 4.7 million kilometers at the furthest extent of the galactic disk. Since the speeds of the star systems orbiting the galactic core in the spiral arms of Andromeda is known while the orbital speed due to the normal gravitational attraction (F=mg) to the core would normally go to zero at the furthest extent in the disk, the actual observed orbital speed of the furthest star or star system in Andromeda would be generated by a half-light year's worth of dark energy. Now given the actual width of the three-dimensional sheet in units of dark energy, a simple proportion could be used to calculate the theoretical amount of dark energy (gravitational vector potential) in each point in space and that compared to other measurements to confirm the model

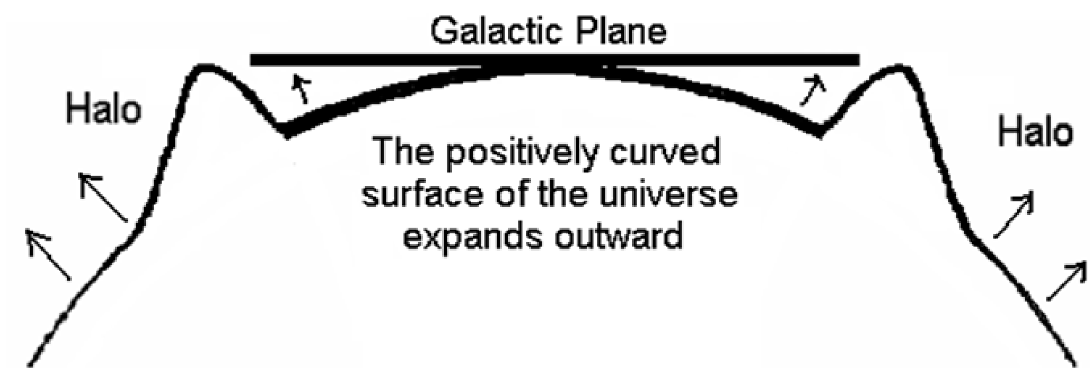
A problem might arise with this method in those cases where a singularity or black hole is present at the center of a galaxy. A mathematical singularity corresponds to a geometrical point of location that has infinite curvature, corresponding to infinite material density. In this case, the shadow curvature in the halo might be distorted because it could not perfectly mirror the infinite curvature of a mathematical singularity. However, an interesting possibility for a solution to the problem also arises from this model. The halo curvature would need to compensate in some manner for the net effect of the singularity within its own curvature. Yet how could a finite shadow of curvature result from the infinitely extended curvature of a singularity? The answer is simple. A physical singularity such as a black hole is not necessarily equivalent to its mathematical model since the real fourth dimension of space is macroscopically extended but closed with respect to the normal three dimensions of space, as specified in the modified Kaluza model.

Einstein never believed in or accepted the real possibility of black holes that were associated with mathematical singularities. It was not especially the black holes that stumped him, but rather the existence of singularities in real physical space. But it now turns out that, at least in this unified field model, mathematical singularities do not exist because the higher embedding dimension is closed. Black holes can, however, and do exist as extremely massive collapsed stars from which light cannot escape. Black holes of this type have nothing to do with mathematical singularities and were predicted from Newtonian gravity they by John Mitchell and Simone de Laplace more than a century before Einstein developed general relativity. The modern concept of black holes could still correspond to physical singularities that could have some of the same characteristics as mathematical singularities without being infinite. So the notion of black holes must still be rethought to correspond to modern astronomical observations. In other words, a physical singularity cannot have an infinite curvature as specified by the mathematics, although the real curvature does approach a zero value without ever having reached it for the diameter (the singularity limit) in three-dimensional space at an extremely large but finite extension in the fourth dimension. This bit of information should lead to a new view of black holes that could result in a better understanding of their physical nature and internal structure.

Three more predictions from the model

Any physical model or theory is only as good as its usefulness for calculations and its ability to make verifiable predictions. This model is no different. However, unlike other models and explanations of DM and DE, this model makes several testable and easily verifiable predictions. Three of these predictions are of immediate interest. In particular, this model predicts that the expansion rate of the universe is undergoing a period of increase. The increase occurs during the mature and old age phases of a galaxy's lifetime. Conversely, the expansion would have been decreasing or slowing at an unprecedented rate during an earlier period of the past history of the universe corresponding to the galaxy-building era or phase since galaxy building absorbs dark energy from the surrounding overall spatial curved universe.

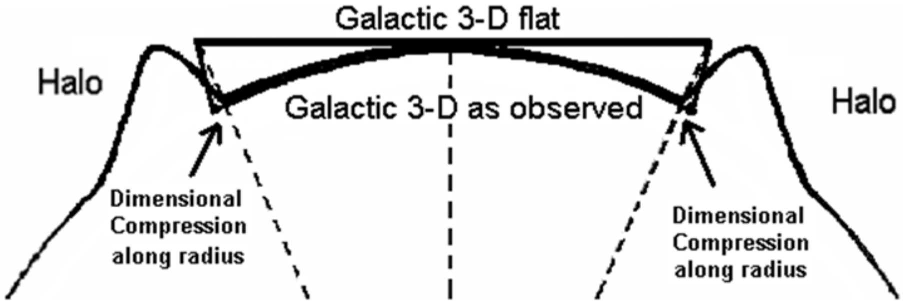
Quite beyond any questions whether the expansion will ultimately stop and reverse, continue forever, or stabilize, the standard model of cosmology based on normal general relativity assumes that the rate of expansion is roughly constant. Within this context, the Einstein radius is increasing and the universe is moving toward a flatter overall curvature over time.



As the universe expands, the theoretical dimensional gap between the positively curved surface of the universe and any given spiral galaxy is decreasing or closing. In reality there is not dimensional gap, it is merely a theoretical artifact. However, the start systems in the spiral arms along the galactic disk still have an excess or disproportional amount of dark energy that propels them at roughly equal orbital speeds around the galactic core. So, while the dimensional gap does not really shrink as the rest of the curved surface of the universe expands out to meet it, as pictured, the excess dark energy equal to the imaginary dimensional gap does bleed back to the rest of the universe as the gap theoretically closes due to expansion. The gravitational potential energy that is normally derived from this dimensional gap is slowly leaking away and slightly speeds up the overall expansion of the universe. As the dark energy returns to the universe as a whole, galaxies are moving apart at a slightly elevated rate that is increasing and will tend to increase for a while, but not forever. According to the conservation of energy, this energy potential or curvature must return to the universe as a whole and thus increases the expansion rate of the universe while doing so. If the number of galaxies in the universe, their average radius and mass distribution were truly known, the amount of dark energy (gravitational vector potential) returning to the universe could easily be calculated and compared to the observed increase in expansion rate. However, given the fact that it is virtually impossible to know exactly these physical parameters of galaxies, approximate figures could be calculated by reversing the process from the known measured value of the present rate of expansion and how much it is increasing.

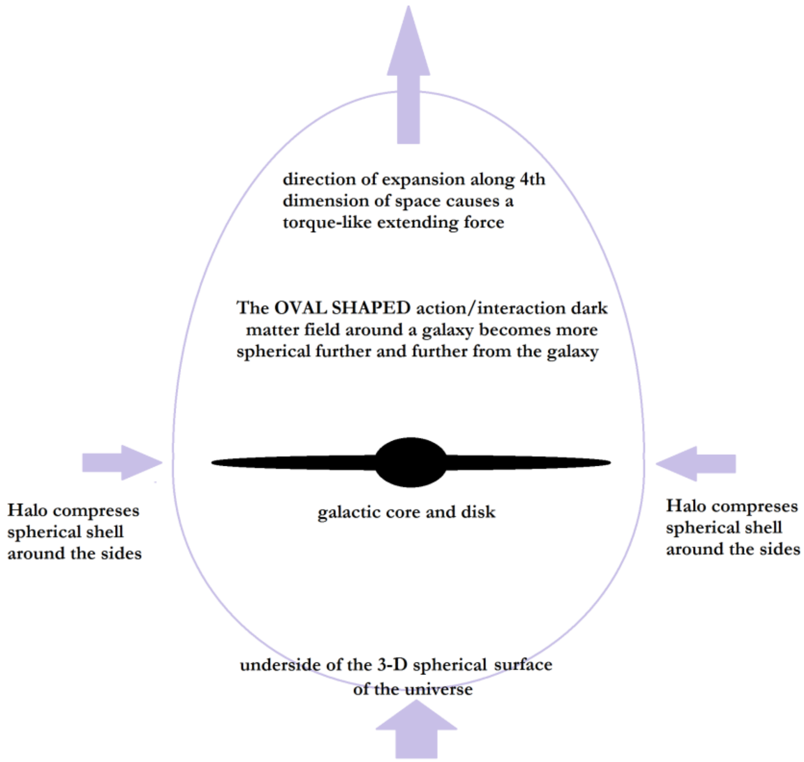
The same methodology could also be applied to calculate the dark energy content in the spatial vacuum between stars, galaxies and other material bodies. Since the Pythagorean Theorem can be used to calculate the 'height' of the dimensional gap between the galactic plane and the global curvature of the universe at any point in space or space-time, this calculated discrepancy could be expressed in energy units when compared to the amount of gravitational vector potential energy supplied by the discrepancy. The potential energy supplied to star systems in the galactic rims would be directly proportional to the linear measure of discrepancy in curvature and thus to the amount of dark energy present. Then, given the 'effective width' of the vacuum or free space in the fourth spatial direction, our little three-dimensional sheet of material reality, a simple proportion would yield the dark energy content of vacuum space, a predicted quantity from this model. This value could then be compared to the accepted value of 10^{-26} kg/ meter^3.

Furthermore, the diameter or radius of any given galaxy should be slightly less than that expected when only gravitational sources for galactic rotation speeds are taken into account. In other words, the disk width of any given galaxy should be slightly less than the expected width according to normal gravity considerations. The amount of this discrepancy could easily be calculated through simple geometrical triangulation since this form of 'galactic radius compression' is predicted by the geometry.



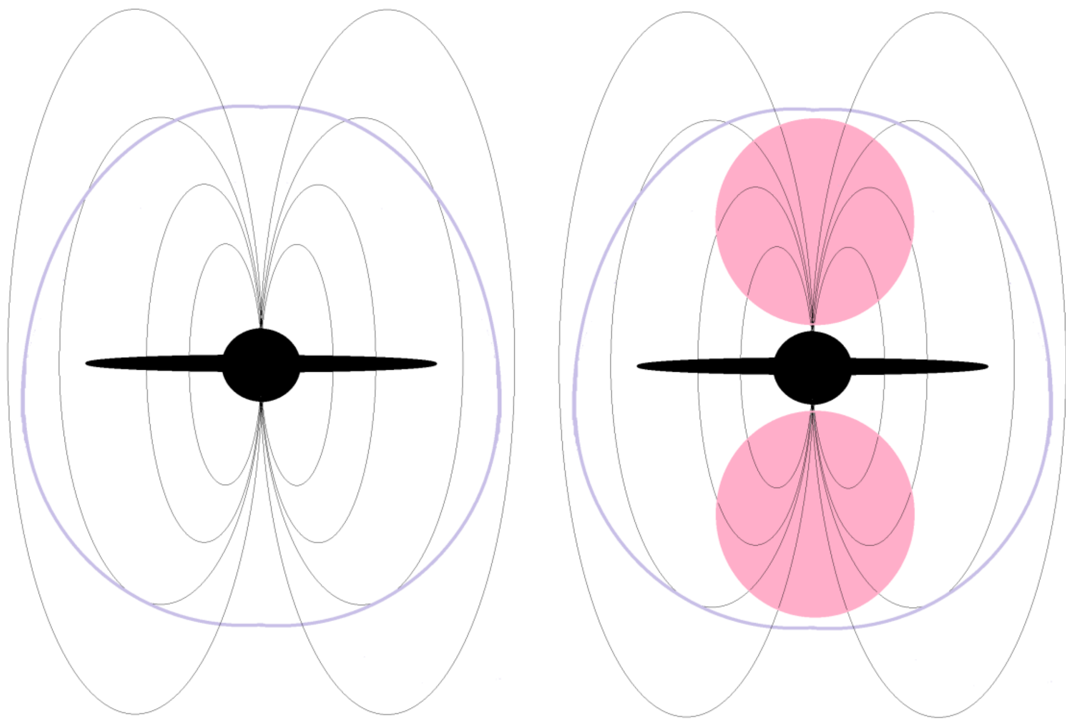
As the Λ-CDM halo 'appears' to 'pull' the three-dimensional flat galaxy 'down' to the three-dimensional curved surface of the universe, a small foreshortening of the galaxy along the radius or diameter would appear from the three-dimensional perspective of observers constrained to the three-dimensional surface. Once again, the actual amount of this visual field compression of the galactic disk diameter can be calculated for any given galaxy using the Pythagorean Theorem and simple geometrical considerations and then compared to observation. The above predictions offer enough new physics to render this model easily 'falsifiable'. All that is needed to make the calculations are the correct values for the necessary variables from astronomical observations, yet more can still be gained from this model.

The shape of the halo around galaxies could also be predicted. Technically, a sphere or even concentric spheres of the possible dark matter action/interaction field surround any given gravitating body. These spheres are merely potential orbits along which real orbiting bodies could move. However, in the case of galaxies with spiral arms, what seems to be the galactic norm, these spheres are modified by the presence of the orbiting star systems to yield the halo outside the galactic plane or disk, but also more of a 'roughly' oval dark matter action/interaction field shape all around the galaxy.



The normally spherical shell around a centrally orbited object would be compressed along the sides due to the presence of the galactic Λ-CDM halo. It would also be elongated in an upward direction due to spin in the fourth expanding direction of space and somewhat bunched up along the bottom of the galaxy which lies along the underside of the spherically curved universe as a whole. The further away from the galaxy one goes, the closer the action/interaction dark matter field moves toward approximated spheres again. However, these spheres are potential while the oval shaped dark matter field pictured is made real by the presence of material bodies orbiting the galactic core.

The oval shaped dark matter field would be accompanied by normal three-dimensional magnetic field lines surrounding the whole galaxy, depending on the magnetic properties of any given galaxy. Taken together, these two field structures would shape and control the flow of ionized particles in and around a galaxy. This structure could account fro the presence of Fermi-bubbles that have been detected above and below the galactic plane of some galaxies, but each situation would be unique for any particular galaxy.



The Fermi-bubbles would just consist of ionized particle plasmas (probably protons) that are the byproduct of stellar and interstellar interactions. They would tend to collect together above and below the galactic disk as shaped by the combined magnetic and gravnetic fields. The magnetic field would affect the ionization characteristics of the particles and the gravnetic field would affect the material structure of the fluid plasma.

And finally, jets of ionized particles are known to be ejected along the magnetic dipole radial axes of galaxies, forming a pinwheel-like structure with the rotating spiral arms of the galaxies. In some cases the ionized particles ejected along these central axes are moving far faster and more energetically than the magnetic fields can account for. The extra speed and energy come from the gravnetic potential field which propels the material portion of the particles down a steep gravitational gradient as they exit the galaxies. The combined magnetic/gravnetic accelerating gradients would tend to boost the effect of each other and protons undergoing this acceleration would leave galaxies as some of the fastest moving and highest energy objects (ultra-high energy cosmic rays) in the universe. This particular gravnetic effect is quite similar to the sling-shot effect detected by NASA for satellites accelerating around massive objects to gain more speed for space exploration. The same would be true for any accelerated object, whether man-made or not, leaving the local environs of the solar system, as for example the Voyager satellites.

In summary, no physical theory or model can be considered truly scientific if it is not falsifiable and does not yield predictions that can lead to verification. This theory and the DM and DE model derived from it are subject to this restriction. In fact, this model yields several predictions, any of which can be used for verification.

- Since normal matter in galaxies is the source of the DM halos, similar halos would exist around all material objects – galaxies, star systems, individual stars, planets and particles. When the halos for all of the planets in the Solar system are accounted for, it is believed that the Titus-Bode Law can be derived from theoretical considerations.

- The shape of these DM halos would be spherical around individual objects, but approximately elliptical around galaxies. Since spiral galaxies are extended outward roughly in the shape of a flat disk and the extra component of rotational velocity comes from DE, the semi-minor axis of the elliptically shaped halo would correspond to the radius of the spiral shape.
- Since all material bodies have halos, a smaller material body moving around another material body in a parabolic or hyperbolic orbit (following a slingshot path) would experience a very small correction (addition) to its outgoing speed as it moves down the potential gradient. In other words, the orbiting body would pick up DE as it moved away from the central object that it moved around, adding to its normal gravitationally induced speed.
- The Solar system and any similar star system should have a small but noticeable halo around it which would tend to be almost spherical or only slightly elliptical.
- Any material object moving away from a central gravitating body or system of bodies would experience an increase in speed due to its moving down the gravitational potential of the halo. In the case of high speed material charged particles (cosmic rays) accelerated by the magnetic fields around galaxies, black holes and similar objects, extremely high energy cosmic rays would result, at least much higher than electromagnetic forces alone could account for.
- This model predicts an increasing rate of expansion of the universe during the old age of galaxies (the present) as well as a corresponding decrease in the rate of expansion during periods of galactic formation in earlier times. As the universe expands, the dimensional gap between the global curvature of the universe and the galactic plane in four-dimensional space decreases, allowing DE to return to the universe as a whole. This pushes the expansion rate faster and faster. If the total number of galaxies in the universe, their approximate masses and approximate radii were known, then the rate of increase in expansion could be determined from this model and compared to the measured rate of increase.
- The existence of a proton-electron dominated universe is assured, explaining why there are very few anti-particles left over from the Big Bang. If the Big Bang caused the three-dimensional universe to expand internally as well as expand into the higher fourth dimension of space, then the global surface of the universe would act in a manner similar to an expanding three-dimensional bubble with a two-dimensional surface.
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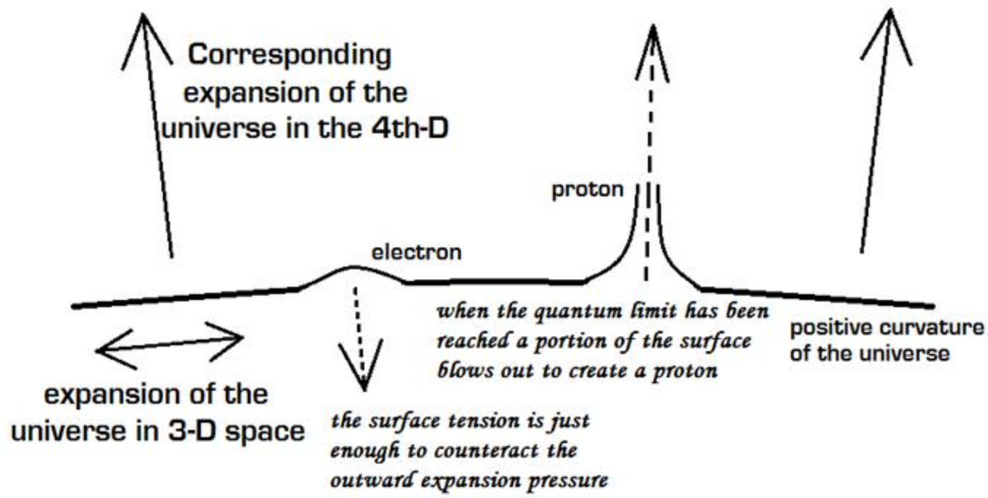


No anti-particles (oppositely directed bumps and hills) are created while the creation of protons and electrons slows expansion enough to end the inflationary period

When the single field density of the surface reached a specific minimum value determined by the quantum and speed of light, individual points in space would have erupted outward (blown out not in) along the direction of expansion in four-space creating protons. During this initial inflationary event, no anti-particles could be created because anti-particles curve inward toward the center of the expanding bubble and against the 'momentum' of the outward direction of the expanding universe. The rapid rate of expansion would slow drastically with the blowout or creation of particles, ending the inflationary period. If the creation of protons were not enough to slow the rate of expansion down or if the energy in the expanding bubble was still too great, a second blowout would have occurred. This blowout would not have had enough energy to create more protons because the surface tension (or energy) of the universe was enough to counteract the blowouts, point-by-point, leaving small bumps in the surface - electrons. The period of cosmic inflation would certainly have halted at this time and no anti-particles would have been created since the motion of the universe expanding outward created the outward protrusions of curvature that are protons, electrons and possibly neutrons.

- The amount of DE in any three-dimensional volume of empty space can be calculated from primary data obtained from the excess speeds of star systems orbiting galactic centers by simple geometrical considerations if the radius of the universe is known. This value could then be compared to known values for the DE content of empty space.
- Galactic diameters should be slightly less than predicted by normal gravity theory. In other words, the diameter (or radius) of a three-dimensional galaxy along its plane surface extended in four-dimensional space would be slightly greater than its corresponding arc distance along the three-dimensional global surface of the universe. Both could be calculated and compared given accurate masses, radii and such data for individual galaxies.

VI. Scientists still need their equations

The stress-energy tensor in the four-dimensional space-time continuum is actually derived from a single point tensor in a six-dimensional manifold.

$$\mathbf{T^{i}_{jk} = S_{6D}}$$

This manifold is as yet physically undefined other than it exists as this point, at least. The use of a six-dimensional field is also not new. Edward Kasner calculated the effects of gravity in a six-dimensional flat solar field, his interpretation of Einstein's general relativity, and published his results about a month before Kaluza's five-dimensional unification appeared. Kasner's model was not, however, a unification model. In this six-dimensional model, the tensor splits into two parts as it drops down into its five-dimensional space-time continuum configuration: a symmetric component representing extension along the fifth direction of space-time and a non-symmetric component representing a point along that direction. The point manifests in the single field as a weak force (gravito-gravnetism) and the extension as a strong force (electromagnetism). These two tensors then drop down to our common four-dimensional space-time continuum where each splits into a symmetric component (electricity and gravity) and a non-symmetric component (magnetism and gravnetism). Our four-dimensional space-time continuum is thus characterized by two natural forces, each having a symmetric (metric or extension) component and a non- or anti-symmetric (point) component. This yields the tensor equations that represent electromagnetism and gravitation, respectively.

The general tensor equation, in its six-dimensional configuration, can also be written a bit differently. The four-dimensional space-time continuum, or material world of our common experience, is still a breakdown of the six-dimensional tensor, but in this case three constants restrict or define the single field occupying the five-dimensional space-time structure: Planck's constant h, magnetic permeability μ and electric permittivity ϵ.

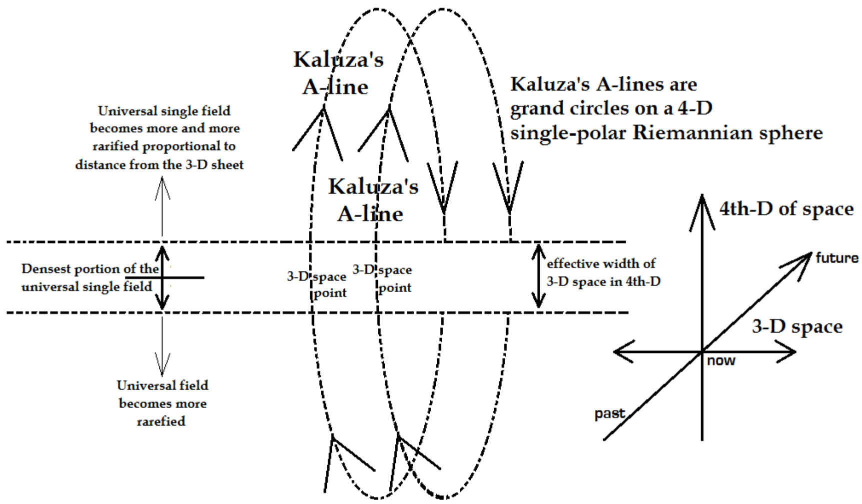
$$\mathbf{T_{4D} = S^{h\mu}_{\epsilon}}$$

Planck's constant represents the binding of space and time to create the space-time continuum, the electric permittivity is the binding constant for the three dimensions that constitute normal space and the magnetic permeability is the binding constant for three-dimensional space to the fourth spatial dimension. Therefore, Planck's constant and the quantum cannot be derived from an over-restriction of the mathematics involved in unifying electromagnetism and gravitation as Einstein believed. The speed of light is a product of the permittivity and the permeability as Maxwell foresaw, such that $c = (\mu_0\epsilon_0)^{-1/2}$. Light and electromagnetic waves thus have two different components: the normal transverse wave in three-dimensional space that Maxwell's theory explained and a corresponding longitudinal wave spreading out three-dimensionally was predicted by as it travels in the fourth direction of space. The prediction of the longitudinal portion of the electromagnetic wave was analyzed and predicted by Whittaker (1903, 1904) but never observed or confirmed.

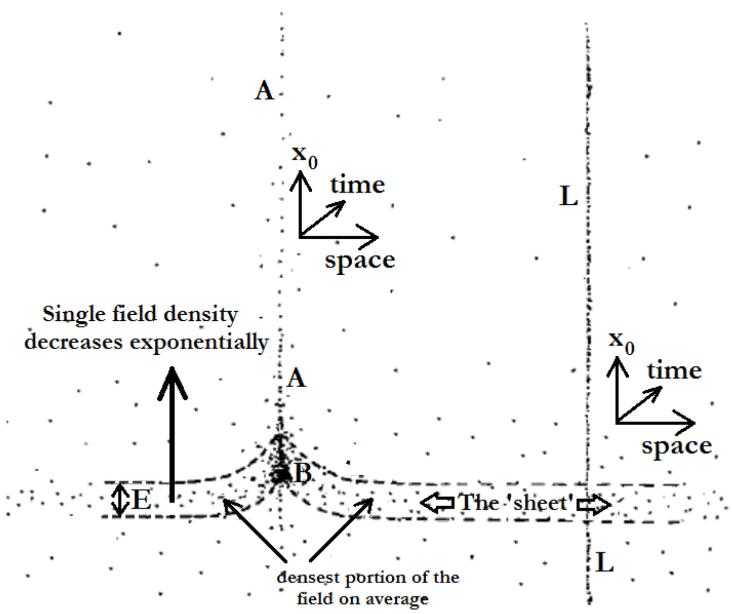
The two tensors that represent the ingle field in the five-dimensional configuration basically determine the density structure of the overall five-dimensional single field.

$$\mathbf{T}^{i}_{jk} = \mathbf{E}_{5\text{-}D} + \hat{\mathbf{G}}_{5\text{-}D}$$

The density variations in the five-dimensional field structure then manifest as the four-dimensional space-time ‘sheet’ that constitutes our material reality. The ‘sheet’ is merely the densest three-dimensional portion of the five-dimensional single field. In general (except where particles occur), the single field density decreases exponentially as the distance from the ‘sheet’ increases either above or below the ‘sheet’ in the fourth direction of space.



This ‘sheet’ is the portion of the three-dimensional space that curves extrinsically into the higher embedding fourth dimension of space (fifth of space-time) to form the hills, bumps and burble that we sense as matter.



This diagram merely provides a means of vizualizing space-time curvature in the fifth direction x_0 of space with the time coordinate passing into the page. The curvature of space follows the densest part of the field as represented by dots. The dots are not meant to represent, in any way, a particle nature of the field, space or time. B represents the field cusp at the center of the particle where the curvature folds onto itself to create the particle due to the 'effective width' E of the space-time continuum. A is the line of symmetry defining the wave-particle nature of matter while L is the represntation of a photon in the field structure. The photon L has a cross-sectional width across the 'sheet' of space-time, which gives the photon its particle-like characteristics.
The dotted line shown above, which outlines the curvature of the 'sheet' in the 3-space direction does not really exist. It is merely a way of depicting the quantum determined limits for the 'effective width' of the space-time 'sheet' in the extra dimension of space.

The various amounts of curvature, once quantized, correspond to protons, electrons and neutrinos. They could also be described as folds in the ‘sheet’ or centers (cusps) or knots of high density that are characterized by effective boundaries determined by the quantum of measurement.

In any case, these particles further differentiate the field according to its average density about any point in space which yields the electromagnetic and gravitogravnetic (gravitational) effects that we associate with matter according to the four-dimensional equation of

$$\mathbf{T}^{i}_{j\,k} = \mathbf{F}_{ik} + \hat{\mathbf{F}}_{ik} + \mathbf{G}_{ik} + \hat{\mathbf{\Gamma}}_{ik}.$$

When Kaluza's cut-transformation is applied to the general equation for the stress-energy tenor, the electromagnetic properties of matter are prioritized and Maxwellian electromagnetism emerges in three-dimensional space. On the other hand, when Kaluza's four-transformation is applied to the general stress-energy/curvature equation gravitation emerges.

In turn, this last tensor equation can also be rewritten in Newtonian terms for a less accurate description of our world as the Lorentz equation and the modified Newton gravity (Heaviside's) equation.

$$\boldsymbol{F}_{em} = q\boldsymbol{E} + m\boldsymbol{v} \otimes \boldsymbol{B}$$

and

$$\boldsymbol{F}_{gr} = m\boldsymbol{g} + m\boldsymbol{v} \otimes \boldsymbol{\Gamma}$$

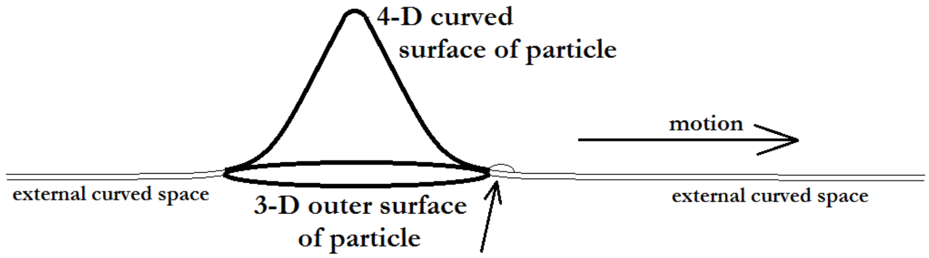
So the Lorentz equation emerges naturally from the dual point-extension structure of the space-time continuum (geometric) as if affects the single field and intrinsic (physical/material) field structures.

$$\Delta \cdot \vec{\mathbf{B}} = \Delta \cdot (\Delta \otimes \vec{\mathbf{A}}) = 0$$

$$\Delta \cdot \vec{\mathbf{\Gamma}} = \Delta \cdot (\Delta \otimes \vec{I}) = 0$$

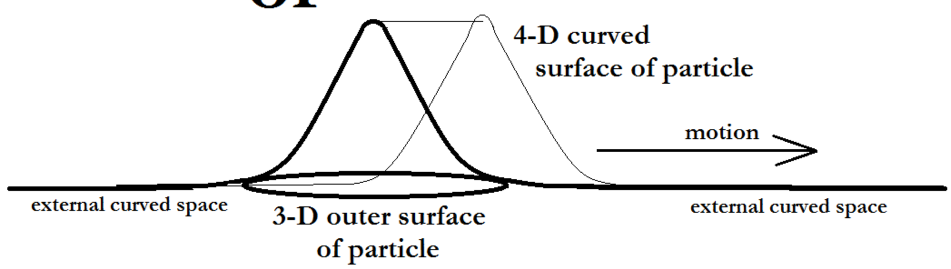
Since the del function or operator (Δ) essentially means to take a derivative in each of the three dimensions of space simultaneously, del dot the magnetic field **B** means that **the magnetic field B itself** is three-dimensional while del cross **A** yields a four-dimensional quantity or rather **A** itself is a vector in the fourth direction of space extending from each and every point in field point in three-dimensional space. The same would hold true for **Γ** which is the three-dimensional continuum or single field component curved in the fourth dimension of space and **I** is a four-dimensional vector. Since the curvature of space-time differs inside and outside of a particle (extreme versus minimal)**,** the gravnetic vector **I** differs inside and outside of a particle. Inside a particle I corresponds to the point-by-point mass inertia of a particle while it corresponds to the point-by-point dark energy (what is called mistakenly dark energy) content of empty space.

In the standard model of the quantum, these would be interpreted as Higgs particles and the Higgs boson field. The so-called Higgs particle is thus easily explained within the unified field theory. Two possibilities exist for explaining how a particle moves through space, which correspond to two possibilities of how a particle is related to or constructed from space-time curvature. An elementary of fundamental particle could either be a quantized and thus fixed portion of the overall curved continuum or a delimited portion of a wave in the continuum. In the first case, a particle would move through the continuum like a boat causing a wake at its forefront. The wake would contribute increased the inertia of the particle (the inertial points absorbed by the particle would raise the overall internal that constitutes the particle higher and steeper in the fourth dimension) as each point in the wake (external space) is overrun by the advancing particle/boat. This would be similar to Higgs' analogy for describing how a Higgs boson is transferred to a particle from the Higgs field to increase the mass of the particle while the particle moves point-by-point through space.



as the particle moves to the right relative to the external space-time curvature it creates a point-by-point wake at its front edge that contracts the particle in the dimension of travel and adds point inertia $\vec{I}$ from external space (Higgs like) to increase its internal overall curvature and inertial mass

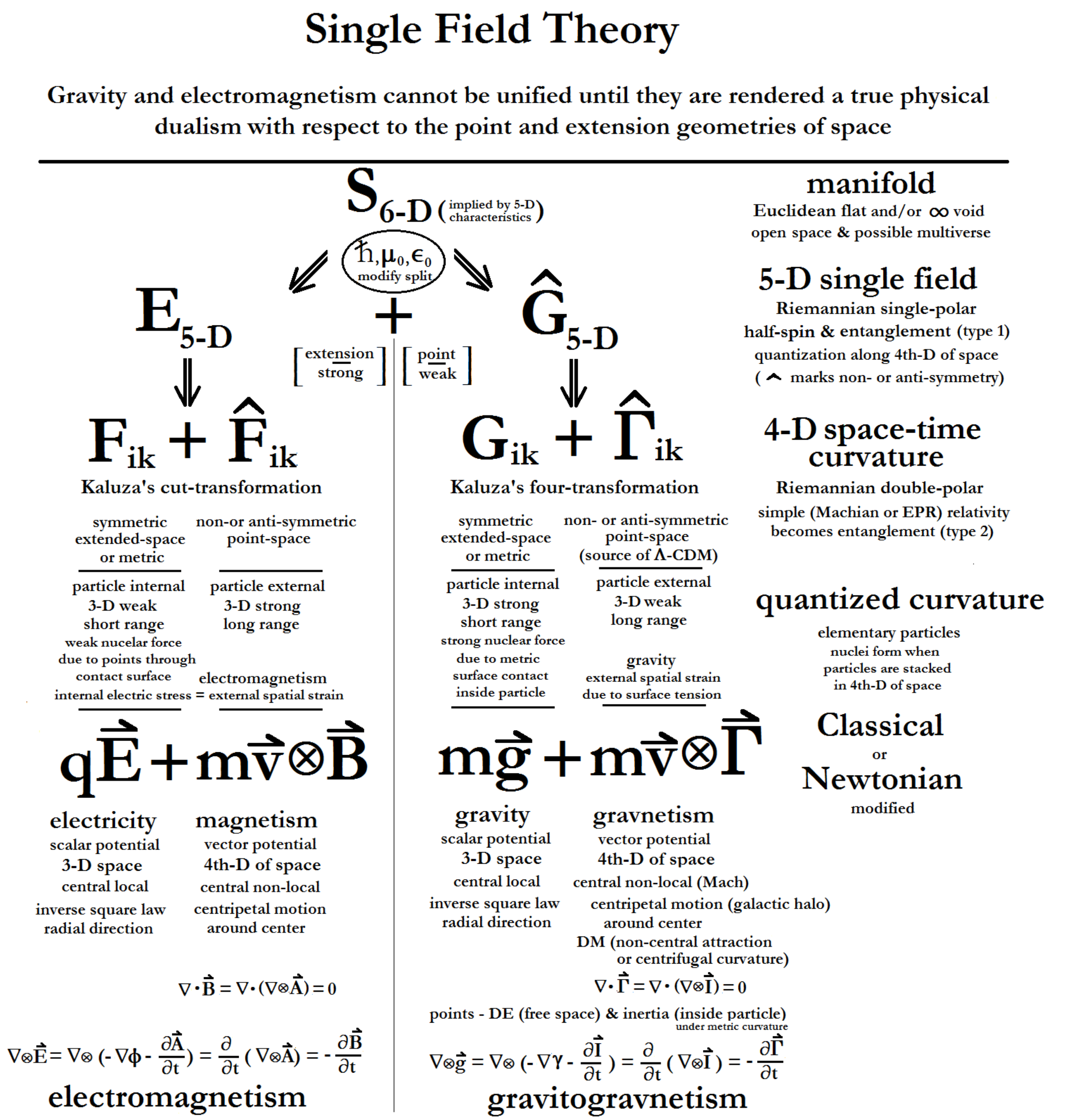
or



the particle/wave moves forward like a ripple through space-time with its central curvature increasing as it passes through each point of external space absorbing point inertia $\vec{I}$

Otherwise, a particle could move through space like a ripple or wave within the overall space-time curvature of the continuum. In this case, the delimited peak of the wave that defines the particle by its width would increase in height in the fourth direction of space as its inertial mass increases due to absorbing more points as it moves (ripples) through the curved space-time. I either case, there would be a pressure of a sort at the forward surface or bow of the wave/particle that would Lorentz-Fitzgerald contract the particle in its width in the direction of travel.

In summary then,



VII. Quantized curvature

The quantum nature of the universe is reflected in the first equation as a fundamental property of the space-time continuum, but exactly how to apply the quantum concept to the single field and space-time curvature still needs some explanation. Klein had the right idea when he tried to quantize Kaluza's five-dimensional extension of general relativity. If you quantize the extension along the fifth direction of space-time (or the fourth direction of space), then the lower four dimensions of space-time (or three dimensions of space) will also be quantized. However, since Klein was working with the Kaluza model, which was vastly over-restricted in its application of a higher embedding geometry, Klein's model was subsequently over-restricted. The fact that what happens along the extra dimension of space also affects the lower three dimensions of space is also implied in the Einstein-Bergmann model of Kaluza's work, although they did not apply this to the quantum as did Klein. Instead, Einstein added a footnote that he and Rosen had attempted to explain particles, unsuccessfully, on the basis of the Einstein-Rosen Bridge.

If instead, we go back to the Heaviside equation or its modern equivalent, quantization becomes a very easy and telling task.

$$\vec{\mathbf{F}}_{gr} = m\vec{\mathbf{g}} + m\vec{\mathbf{v}} \otimes \vec{\boldsymbol{\Gamma}}$$

$$\text{but since} \qquad \lambda = \frac{h}{mv}$$

$$\vec{\mathbf{F}}_{gr} = m\vec{\mathbf{g}} + \frac{h}{\lambda}\vec{\mathbf{r}} \otimes \vec{\boldsymbol{\Gamma}}$$

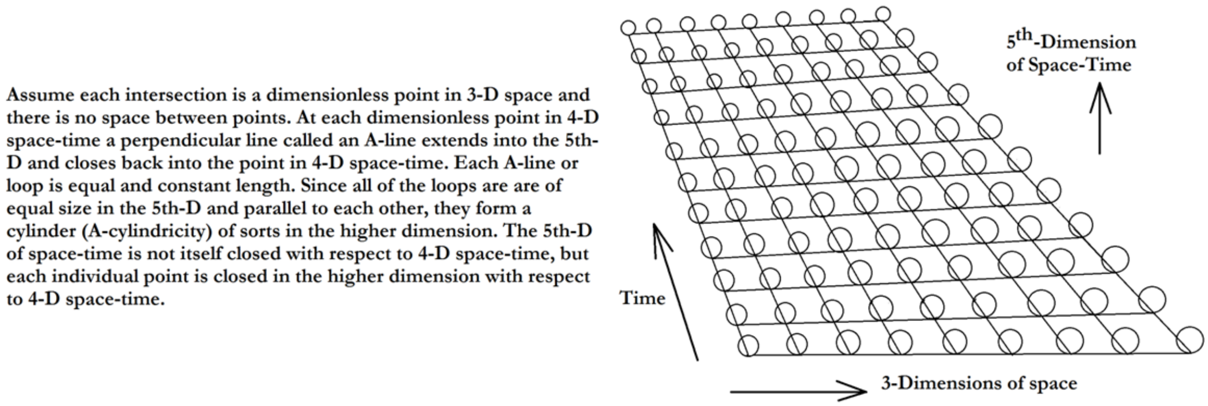
This result reflects quantization on the cheap. While only an approximation, the result does have significant meaning. First of all, normal gravity (**F**=m**g**) as well as Einstein's original version of general relativity ($\mathbf{T}_{ik}$=$\mathbf{G}_{ik}$) is about an extension-geometrical version of gravity, in other words, it is about a static gravity only universe, just a fixed picture at a moment in time of a dynamical universe. Variational and other mathematical tricks need to be applied to this gravitational picture of the universe to determine how gravitational attractions evolve over time. However, the second term of gravity introduces a dynamical term to gravity surrounding a point-geometry that indicates a quantum perspective of the world. The unit vector **r** has been added since this new dynamical form of quantized gravity must be expressed in polar coordinates, which are more natural to this physical example.

The implications of this simple quantization are quite profound. First of all, the dark matter halo around galaxies, represented by the second term in the equation, can now be viewed as a wave interaction or superposition of outgoing quantized gravity waves from the core mass due to the changes in velocity of the orbiting masses interacting with the incoming gravity wave (represented by **Γ** in "/sec" units). In this case, the quantity **Γ** could be likened to David Bohm's concept of the quantum potential field which is a background field consisting of all possible quantum waves. Secondly, this equation indicates that the static gravity view of **F** = m**g** cannot be quantized in any normal way within the three-dimensional space where normal gravity acts. This implies that gravity can only be quantized within the higher embedding dimension. From a purely static point-of-view, normal gravity is purely structural in our world while the quantum immediately invokes an interaction which is wholly dynamical and non-structural except in the fact that it changes structure through the interaction.

And thirdly, this confirms the interpretation of the quantum as a point-geometric version of space rather than truly discrete unconnected physical and thus indeterministic reality. In other words, individual quantum-point phenomena can be considered independent (and thus discrete) of the overall curvature of space-time as if they are matter waves moving across the background of a fixed space-time curvature. Quantum phenomena are dynamic point by point interactions that occur against the backdrop of curved space, but in order to understand them they must be integrated into the overall curvature of the space-time continuum to make any sense. With these new clues of information in hand, a more comprehensive view of curvature and particles can be rendered.

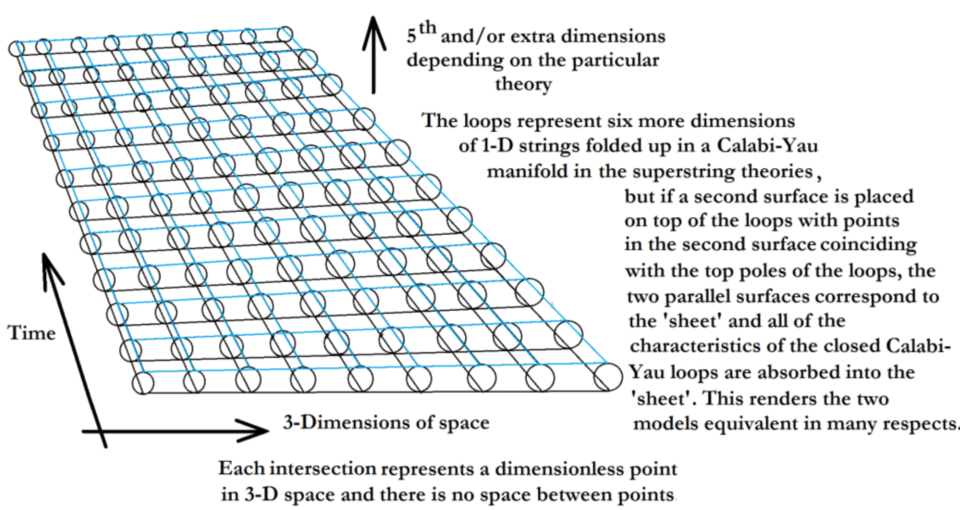
Quantizing surfaces point-by-point into the single field

In the Kaluza-Klein model, each discrete point in space is extended in a one-dimensional quantized loop extending into the fifth dimension of space-time.



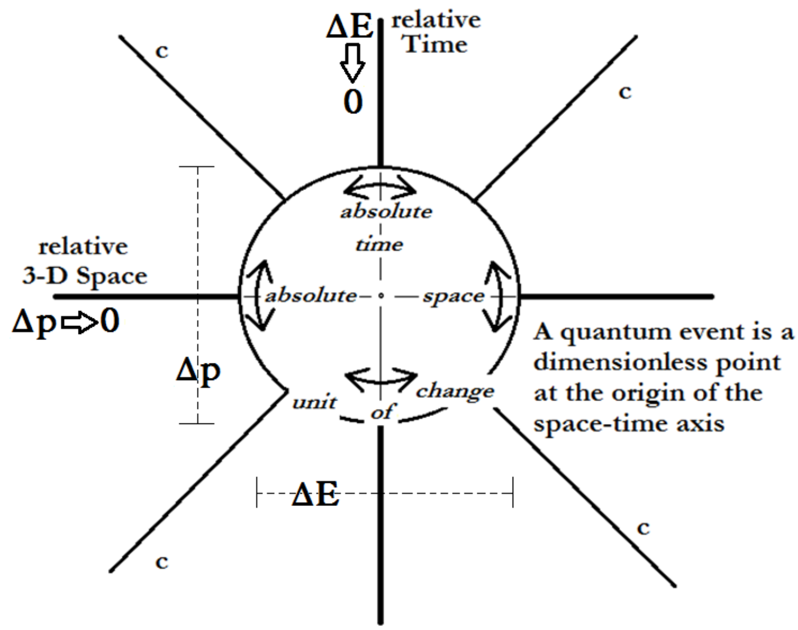
Each loop is not connected to the other loops outside of the normal three-dimensional point-to-point connection mistakenly assumed and thus afforded by the intrinsic nature of the space-time continuum. In other words, Kaluza utilized a higher embedding dimension, but mysteriously retained the intrinsic nature of space-time curvature. Thus he used a fundamentally flawed mixed intrinsic/extrinsic model of physical reality. Klein's idea that quantification in the fifth-direction of space will quantize the lower four-dimensional space-time is valid, but only under the newly specified geometrical conditions that render our normal three-dimensional space real.

If the loops are made to connect three-dimensionally at all points along their four-dimensional A-line loop extensions, they would instead form a sandwich of a sort with an 'effective width' in the embedding dimension.

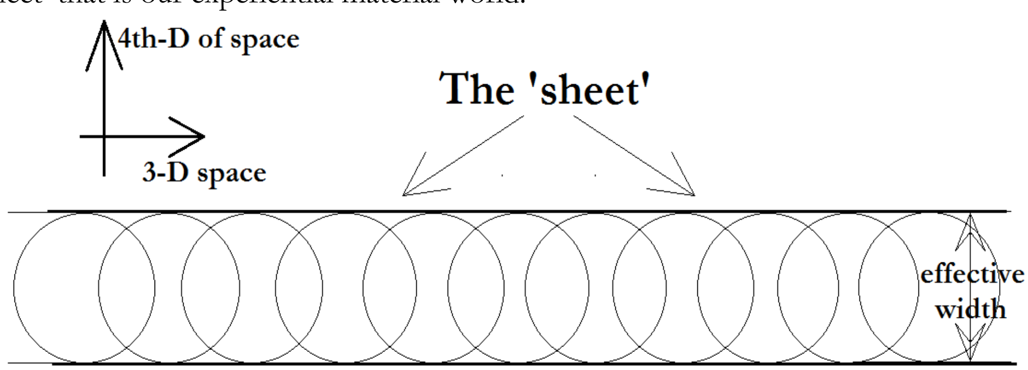


This sandwich of loops, or 'sheet', thus constitutes our normally perceived four-dimensional space-time continuum or rather the primary n=1 space-time continuum. It does not matter whether the A-line loops are one-dimensional as in Kaluza-Klein or six-dimensional Calabi-Yau bundles as in superstring theory. The whole of our three-dimensional space or four-dimensional space-time is literally within this 'sheet', which is extrinsically curved in various ways to produce what we perceive and detect as matter, light and their many manifold interactions. So the whole Kaluza-Klein model as well as superstring theory can be incorporated into this new model as intrinsic properties of the 'sheet'. This should be obvious since the superstring model seems to ignore the curvature of space-time in any of its physical pronouncements even though it is originally based on Kaluza-Klein, which is a curvature model of space-time.

Since the 'quantum of action' can be defined as smallest possible measurement of a quantum 'unit of change' in our material world, the 'effective width' (effective because it has no true fixed width being a continuity) of that 'sheet' would be related to the smallest measured real particle.



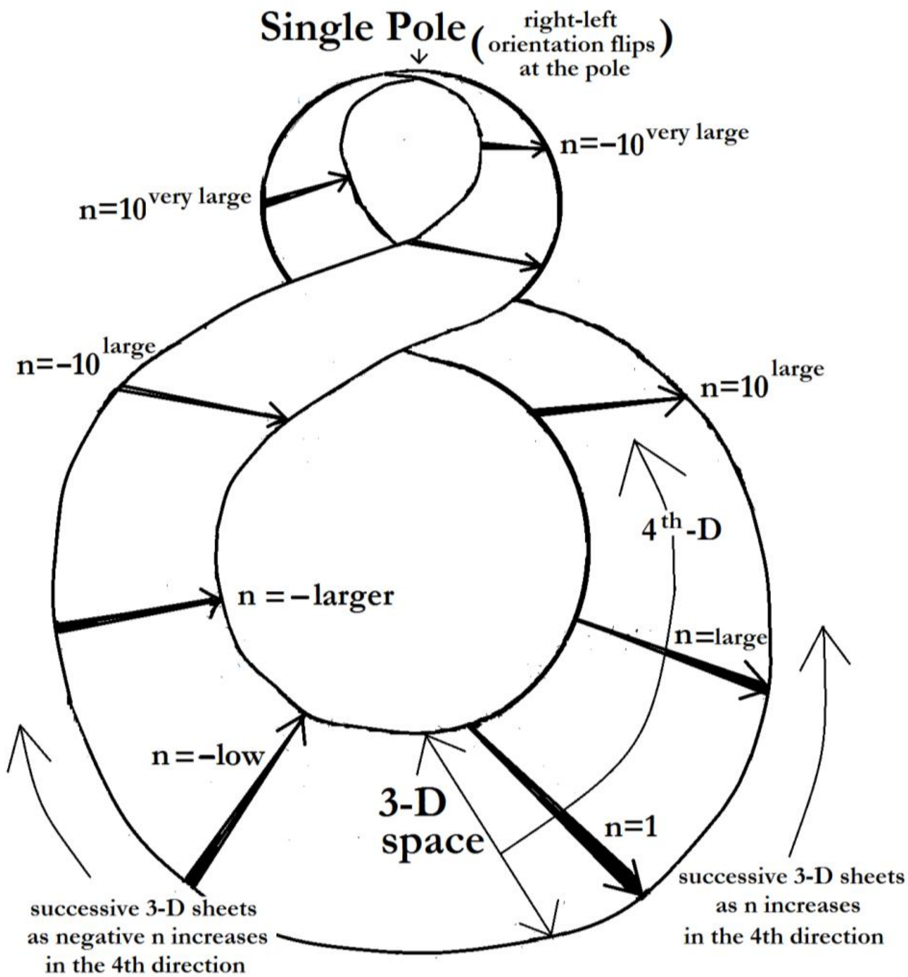
The geometrical points in space or space-time about which the quantum would then be considered action or actionable points in space around which the fundamental unit of change determined the quantized three-dimensional 'sheet' that is our experiential material world.



Fundamental 'units of change' determine the 'effective width' of the 'sheet' when they reach the lower limit of 'quantum measurement'

This quantizes the space-time curvature and unifies the space-time continuum of general relativity and the quantum world. The fundamental 'sheet' where the single field is on average densest would account for the n=1 quantum state or ground state of a static universe defined by its static curvature, $\mathbf{T}_{ik} = \mathbf{G}_{ik}$. However, higher quantum states can be attained dynamically on a point-by-point basis. Higher quantum states, for n=2 to n = nearly infinity, would appear like pages in a book stacked one on top of the other both above and below the primary n=1 'sheet'.

Since the higher embedding dimension is single-polar and all extensions of three-dimensional points into the higher fourth dimension of space pass through the single pole, the universe would look something like a two-dimensional Möbius strip extended and closing into a point where the directions change left of right.



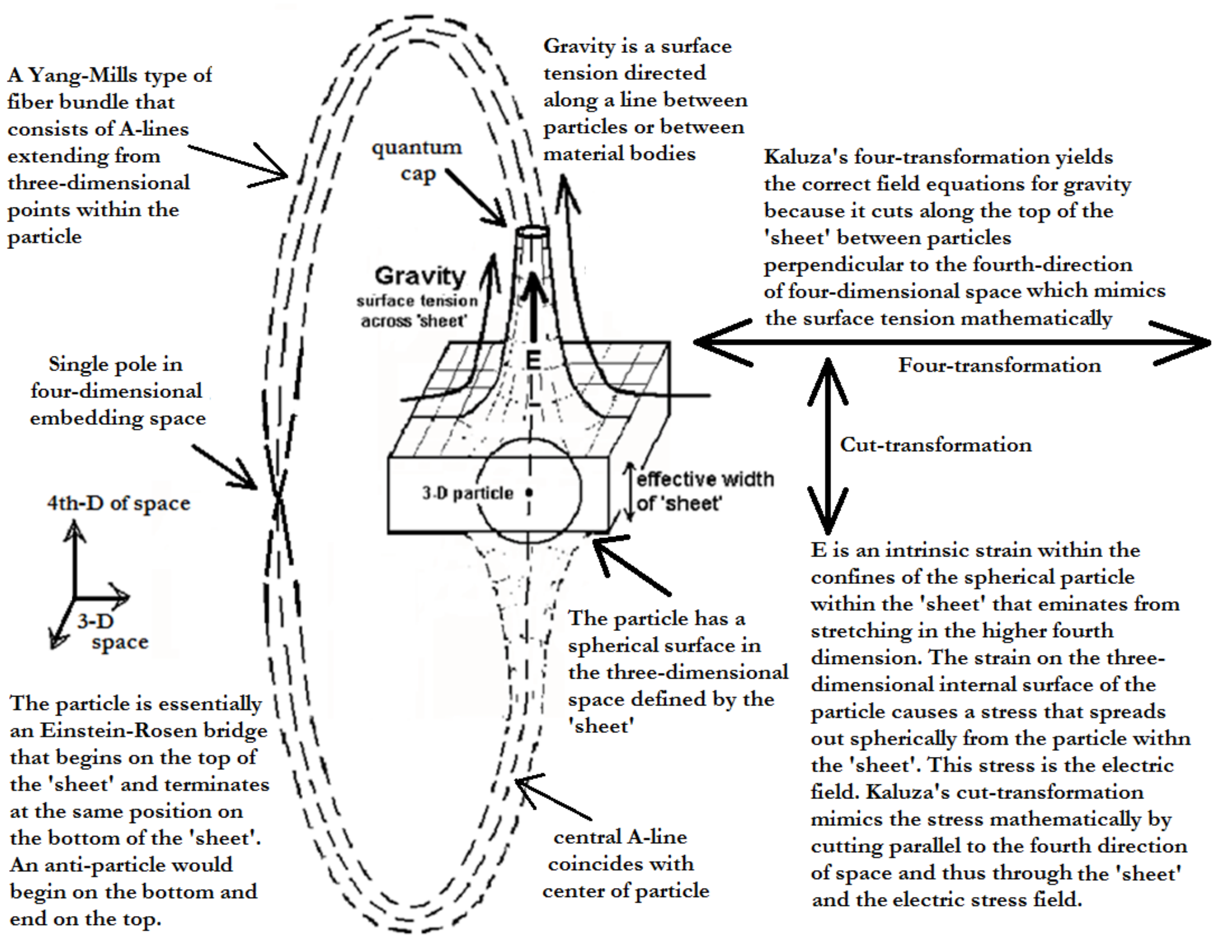
Since the higher dimension is extended to at least as great a distance as the other three dimensions of space (the Einstein radius of the universe), the final quantum number would be nearly infinity in this case. The normal laws and theories of physics that apply in three-dimensional space would be strictly limited to the three-dimensional 'sheets'. However, the only three-dimensional 'sheet' that is characterized by curvature is the primary or fundamental 'sheet' (quantum ground state of n=1) so while the theories and laws of physics influence only three-dimensional extension within the 'sheets', the laws must be modified for non-ground state ‘sheets’, each of which is smaller than the preceding ‘sheet’ as they ‘shrink’ to zero three-dimensional extent near the single pole.

Technically the limit of the speed of light in empty space only applies three-dimensionally so ‘signals’ along the fourth direction of space are instantaneous. Since all real material particles must conform to this geometry, all real particles must have half-spin. Under special circumstances, evidence of this connection between al points in three-dimensional space has been identified as ‘quantum entanglement’ in instances where interacting with one particle causes changes in a distant partner particle. The spin-spin coupling of real particles (as in Pauli’s Principle) would be an example of this type of entanglement. Any so-called quantum particles that do not have half-spin are not real, but instead temporary energy resonances in the single field until they decay spontaneously into some form of real particles with half-spin. These, pseudo- or temporary-particles decay into real particles and photons because they do not meet the strict geometrical and quantum conditions of reality. They can also be connected three-dimensionally through higher quantum-numbered ‘sheets’ where the normal laws of physics prevail, but the ‘sheets’ are less dense so the physical constants of our normal three-dimensional space may not be the same.

Planck’s constant is the binding constant for space and time, while electrical permittivity is the binding constant for the normal three-dimensions of space and magnetic permeability is the binding constant for points along the fourth direction of space or of the three-dimensional sheets one to the other in the fourth direction of space. Since the single field becomes less and less dense the further away from the ground state ‘sheet’, permittivity and permeability can differ within the higher energy state ‘sheets’ where $n>1$, so while the ‘speed of light’ across those three-dimensional ‘sheets’ cannot be exceeded, the ‘speed of light’ itself across those ‘sheets’ may be relatively lower than in the $n=1$ ground state ‘sheet’. Quantum entanglement can thus take place three-dimensionally and still exceed the ‘speed of light’ in the ground state ‘sheet’ by taking advantage of this property in the higher energy state ‘sheets’. Entanglement of some properties of light waves, such as polarity, is of this type.

True material particles are stable and have a spin of $\pm\frac{1}{2}$. These hills (protons), bumps (electrons) and burble (various neutrinos) of curvature of the ground state more-or-less represent folds in the ‘sheet’ that extend beyond the approximate flatness of the ‘sheet’ into the higher embedding dimension. Particles are not points, although they heave centers of mass, and there are no true mathematical singularities within them, even at their centers of mass, because space in the fourth direction is closed. True particles must confirm to the geometrical conditions established by the Riemannian geometry of the embedded space, so instead of singularities at the particles center, the loop or bundle of points that constitute the extended particle in three-dimensional space goes through the single-pole of the higher space and returns to the extended particle on the opposite side of the ‘sheet’ to give the particle and overall half-spin that matches that of the central point.

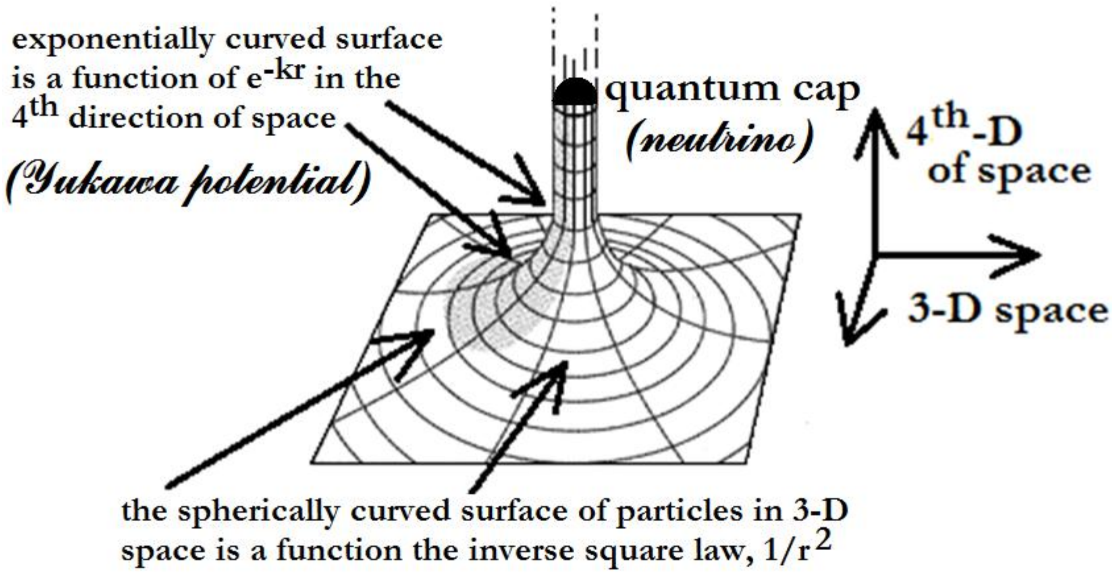
The particle, in this case a proton, would look like an Einstein-Rosen Bridge opening from three-dimensional space and twists around in the fourth dimension to return to itself in the ‘sheet’. However, the particle does not really extend all the way around the fourth dimension but is capped off by a neutrino, the smallest amount of curvature that is possible in the ‘sheet’. This would place a downward stress on the quantum cap (neutrino) toward the ‘sheet’ equally distributed through the internal portion of the particle and ending at the three-dimensional boundary of the particle within the ‘sheet’. This stress, due to the expanded or stretched nature of the single field in the direction of the higher fourth dimension, would cause a strain in the three-dimensional space surrounding the particle that science observes and interprets as the electrical field.



This is why Kaluza's cut-transformation which is the mathematical equivalent to cutting a splice across the three-dimensional 'sheet' perpendicular to the fourth direction (and thus around the particles three-dimensional boundary) yields electromagnetism.

On the other hand, another field of potential or 'strain' would extend throughout three-dimensional space emanating from the outer spherical boundary of the material particle that is due to the expanded or stretched nature of the single field in the higher fourth dimension that appears from particle to particle as a surface tension along the surface of the ground state sheet and thus toward each particle. This potential field, the gravity field, would thus fall out of the equations of state when Kaluza's four-transformation is applied to the single field equation. Since it is only a form of 'surface tension' across the top of the 'sheet', gravity would be quite a bit weaker than its electric field equivalent which extends throughout the 'effective width' of the 'sheet'. In this manner, there are no electrical or gravitational forces within the proton or particle that could act against each other to tear the particle apart. The field potentials only appears at the particle boundary as strains in the surrounding three-dimensional physical space and diminishes as the inverse square of distance from the particle boundary.

Within the particle, the surface of curvature and the point-by-point stretching stress due to expansion in the fourth direction change roles in a sense. The internal particle 'force' due to the surface tension becomes by far the stronger of the two internal particles 'forces'. While a particle is outwardly spherical in three-dimensional space, it is inwardly three-dimensional surface an exponentially curved in the fourth direction of space which would form a strong surface to surface 'force' of attraction with another like particle.



Whereas points through the surface from point-to-point between such bound like particles would be quite weak even though it transmits the point-by-point 'stress' of stretching in the higher expanded fourth dimension of space to the outer surface of the particle in three-dimension space. Obviously, these two internal forces that are the geometrical or structural equivalents to the outer forces of strains associated with the gravity and electric fields are just the strong nuclear force (or field) and the weak nuclear force (or field). This structural switch of roles, whereby the external weak force (gravity) becomes the internal strong force and vice versa, explains exactly how and why the internal weak nuclear force has become associated with the stronger external force of electricity and recast as the electro-weak force in earlier unification schemes. However, a great deal more physics is now implied by the geometrical structure of space-time curvature.

The atomic nucleus resolved

Despite everything that physicists claim to know about the atomic nucleus, they know very little about the internal structure of the nucleus. In other words, there is no single theory of the nucleus, but rather several different theories and models, several of which are mutually incompatible, that explain different aspects of the nucleus and its internal structure. The nucleus, of course, is composed of protons and neutrons, but no electrons are allowed to exist in the nucleus. That would be impossible. Yet electrons can be ejected from the nucleus during decay processes just as a positron can be absorbed by the nucleus in other decay processes. In both cases, these and other particles just seem to appear and disappear at the nucleus' outer surface, which seems to imply that all particles within and created by the nucleus are equally represented at the nucleus' surface. Indeed, high-energy collision tests seem to indicate that the nucleus is perfectly spherical for nearly all intents and purposes, while other evidence indicates that the nucleus has an internal shell structure similar to that of the electrons orbiting the nucleus. Both cannot be true. The nucleus cannot be both perfectly spherical in a sense where the constituents of the nucleus seem to melt together in a big drop of liquid and maintain a specific shell-like structure where the protons and neutrons retain their individual characteristics. These properties of the nucleus are explained by the fluid and shell models and their variants.

On the other hand, Hideki Yukawa made his discovery of the Yukawa potential in 1935. He assumed that there must be a particle exchange within the nucleus utilizing a potential field much stronger than the repulsive potential of the protons for each other. Otherwise, the nucleus would just explode under its own internal repulsive electrical forces. The Yukawa potential (or field) is the first of the three primary ingredients necessary in any new model of the nucleus. Yukawa's original reasoning depended on an analogy with two people being held together by the fact that they were playing a game of catch – exchanging the ball brought and holds them together. He developed the idea that nucleons exchanged particles – mesons – with a particularly strong nuclear field which bound them together within the nucleus against the extremely strong electrical repulsive force between protons. This nuclear field must therefore be something like an electric field only much stronger. He was thus able to write down a simple formula for the field potential and predicted the existence of the meson as the exchange particle.

$$V(r) = -g^2 \frac{e^{-kmr}}{r}$$

The existence of his predicted meson was confirmed in 1947 as the temporary short-lived byproduct of cosmic rays colliding with air molecules in the upper atmosphere of the earth. The key to his theoretical model, however, is best exemplified by his concept of a specialized field that acts like an electric field along with the exchange particle concept. The other clues for a new nuclear model reside in the shell and fluid (tear drop) models of the nucleus, which are mutually incompatible with each other.

The liquid-drop model was first formulated by Carl von Weizsäcker in 1935 and the shell model was introduced in 1949 by Eugene Wigner, Maria Goeppert-Mayer and J. Hans Jensen. The processes of fission and fusion were discovered and outwardly explained leading to the development of the nuclear bomb during the same period of time, but no significant theoretical explanations of decay, fission or fusion came from this or similar technological innovations. The nucleus is seen in quantum mechanics as a potential well form which decay particles within the nucleus do not have enough energy to classically escape. However, through a mathematical process called 'quantum tunneling', quantum mechanics has mimicked the decay process allowing particle-waves without the proper classical energy to escape the nucleus potential well to escape. Most interest in classical nuclear theory seems to have diminished once more in the 1970s as more interest developed in the Standard Model of particles and the application of particle theory to the nucleus. All together, this gives five or more models that are used to explain eh nucleus, none of which leads to a single overall model that can not only incorporate all of the previous results, but can explain why they all work.

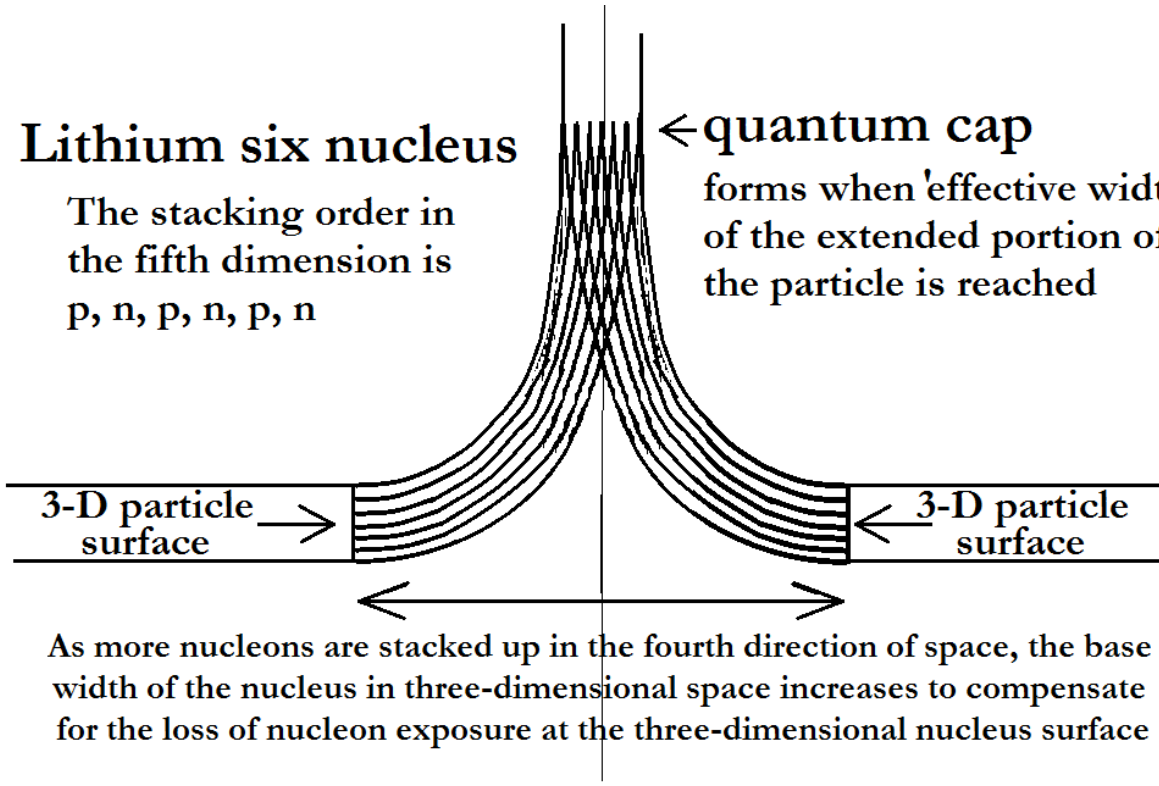
This new nuclear model leaves only the Yukawa Potential and its predicted exchange of mesons for further explanation, both of which can be just as easily explained. The nucleons that form the nucleus are four-dimensional particles yet their individual contact surfaces with the next nucleon above or below (in the fourth direction) are three-dimensional. Therefore, when quantized during radioactive decay or similar processes, the three-dimensional surface of contact quantizes as the three-dimensional meson outside and thus independent of the nucleus. The meson should be considered a mere three-dimensional quantized shadow of the four-dimensional connection (surface of contact) between nucleons in the nucleus, just as three-dimensional objects project a two-dimensional shadow to light. However, the meson as a shadow is more substantial than the light shadow to which it is being compared because it carries away energy as a field resonance pattern in three-dimensional space: It has a semi-material reality due to its energy content rather than a true inertia, at least until it decays. The meson is not a real particle in the same manner as a proton, neutron or electron, because it does not fulfill all of the pre-requisite quantum conditions for particles, *i.e.*, it does not have the ±½ spin that is required by the four-dimensional geometry of space, and thus only enjoys, quite literally, a momentary half-life as a pseudo-particle.

The Yukawa model offers the best possibility of leading to a space-time curvature model of the nucleus. It has already been confirmed to an extent by the mathematical researches of Henry T. Flint and Vu B. Ho. Flint incorporated the Yukawa Potential into his own five-dimensional model of space-time, (Flint, 1946, 1948) while Ho derived the Yukawa Potential mathematically from a general relativistic model of space-time. (Ho, 1995) Similar forays of relativity into the realm of the quantum have been made by Flint and others since the 1920s, but these have gone largely unnoticed by the physics and scientific communities. William Wilson, Flint's teacher before he became his collaborator, derived the Klein-Gordon equation assuming a five-dimensional volume of space-time equal to Schrödinger's wave function in the early days after the origin of quantum and wave mechanics. (Wilson, 1928) Taking Wilson's ideas one step further, a stable material particle can be considered to have a specific four-dimensional spatial volume that must remain constant in time. When the particle moves and thus Lorentz-Fitzgerald contracts in the direction of its motion, the particle compensates by spreading or increasing its height in the higher fourth dimension to maintain its constant volume. The constant volume in four-dimensional space over time is restricted by $E=mc^2$ and thus represents a new formulation of the energy-matter theorem of thermodynamics.

During the period of 1926 to 1966, Flint published more than three dozen articles on his five-dimensional theory in peer-reviewed journals, incorporating every new feature of Einstein and others' work on this subject into his model. Not only did he incorporate the Yukawa Potential into his model, but Flint and J. W. Fisher also incorporated the Dirac equation (Fisher, 1929; Flint and Fisher 1928, 1930) and other quantum features into the five-dimensional field model as he found it necessary. Flint published a summary of this work in 1966 in the book *The Quantum Equation and the Theory of Fields*. No other model of physical reality has come as near to unifying relativity and the quantum as that presented by Flint in his published works, either in the past or the present. Yet Flint and his colleagues' theoretical work has been completely ignored by a scientific community whose attention has been wrongly fixated on a purely quantum solution to unification and physical reality. So there is some mathematical evidence that implies a higher-dimensional model of the nucleus, and indeed that is exactly what fixes all of the problems with previous nuclear models.

Under the proper physical conditions, protons and neutrons can come together to form atomic nuclei. They are then stacked, one upon the other, in the fourth direction of space. This stacking is tantamount to increasing the overall curvature of nucleus to accommodate the new grounds of particles. The stacking in the fourth direction of space is partially compensated by the nucleus spreading out with a larger diameter in three-dimensional space, which evens out the combined curvature of the structure.

The nucleus



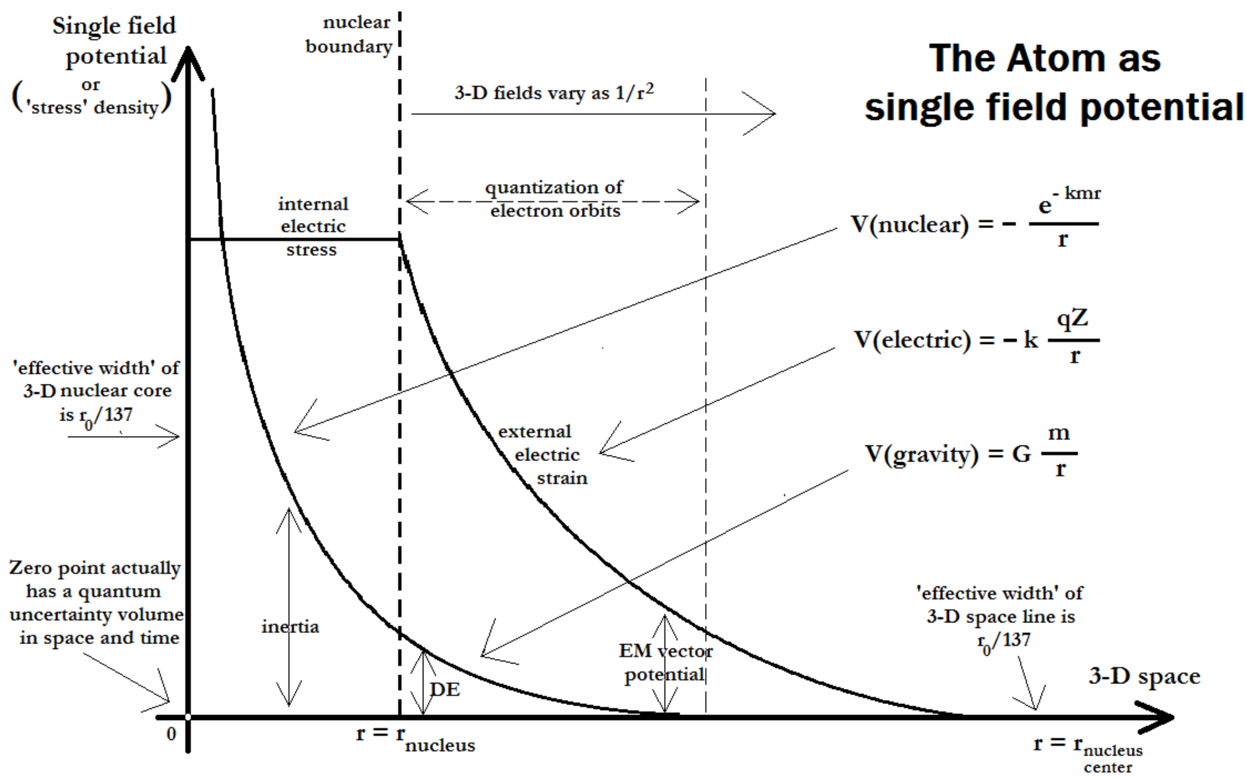
By stacking the nucleons one on the other in the higher extended dimension, the shell model is easily accommodated, while each nucleon would retain some small amount of a presence on the outer spherical three-dimensional boundary of the nucleus. There would thus be no need for quantum tunneling to explain decay while decay products would necessarily appear to be created or absorbed at the surface.

The surface or nuclear boundary in three-dimensional space would remain spherical in nearly all instances and seem fluid. The Yukawa potential is thus merely equal to the four-dimensional surface area of contact between any two contiguous nucleons and no mesons are being exchanged in the nucleus between nucleons in contact. Mesons merely appear in collision experiments, after the fact, because they partially meet the geometric and quantum conditions for real particles, but not completely meeting them they rapidly decay. Exchange mesons could thus be viewed as four-dimensional shells (curvature only without inertia) of pseudo- or temporary three-dimensional particles.

This takes care of the strong nuclear force, but the weak or electroweak force still needs to be explained. The curved surface of contact between contiguous nucleons has the same metric (extension) form or shape as the Yukawa potential, so that part of the nuclear forces is easy to account for. Yet space is also mathematically dual, whether that space is internal to the nucleus or external to it. This means that the point-by-point space inside the nucleus must also be accounted for, which is easy given the explanation of how electrical stress are created inside the nucleus and translated to an electrical field as a strain (the electric field **E**) in the space outside the particle. The point-by-point electrical stress inside the nucleus that comes from a point-by-point pressure (pull) on the quantum cap due to the stretching (expansion) of the single field in the fourth direction of space penetrates the metric surface of curvature between contiguous nucleons point-by-point to create the electroweak force or field. That is why the electroweak force is associated with neutrinos produced during nuclear decay processes. After all, the quantum cap is a neutrino. Given this new information and the five-dimensional model of the nucleus that emerges, a quantized five-dimensional curvature model of the atom as a whole is also strongly implied.

The atom

The additional or anti-symmetric term – Einstein called it a non-symmetric form in his final attempt to develop a unified field theory – represents a point-geometry as opposed to the normal Riemannian metric, which is an extension-geometry. This point-geometry can account for Dark Energy, inertia, the magnetic vector potential and photons, all of which are represented as points in three-dimensional space extended in the fourth dimension of space as one-dimensional lines (Kaluza's A-lines).



In free space these four-dimensional extensions become the magnetic vector potentials, Dark Energy and photons depending on the characteristics of any given situation, but when packaged under the metric curvature surface they become inertia and the stress source of the magnetic field strength that manifests in either moving or stationary particles. If a five-dimensional space-time continuum representing the single field is used to interpret Einstein's equation then no additional term is necessary for Dark Matter, since it is incorporated into the anti-symmetrical portion of the complex tensor. The additional anti-symmetric term also results in a 'twist' in each dimensionless point in three-dimensional space as opposed to the symmetric term of the tensor which represents the normal Riemannian metric or extension-based geometry of space-time curvature in the higher embedding dimension of space.

The point-geometry of the anti-symmetric portion of the tensor accounts for Dark Energy, inertia and the magnetic vector potential as well as photons. The local curvature of the three-dimensional 'sheet' in the fourth dimension is interpreted as gravitational mass, but dimensionless points in three-dimensional space that are extended in the fourth direction of space under the curvature collectively constitute the inertia mass. Those same dimensionless points in three-dimensional free space outside of local particle curvature yield Dark Energy. As a transverse electromagnetic wave front, which is continuous, spreads out in free three-dimensional space according to the inverse square of distance, its movement through three-dimensional space causes magnetic vector potential (four-dimensional) and electric scalar potential (three-dimensional) variations at each dimensionless point in free space. However, the vector potential variation must have magnitude and direction, neither of which can possibly exist within dimensionless points in three-dimensional space, so the vector potential must therefore exist along the four-dimensional A-lines that correspond to points in three-dimensional space.

As the electromagnetic potentials vary at each point in normal free space due to the passing of the transverse wave, the variations that they cause at each point is transmitted in the fourth direction of space as corresponding longitudinal (pressure) electromagnetic waves. The concept of longitudinal electromagnetic waves was developed in the early twentieth century by Edmund T. Whittaker, but they were never detected, nor could they ever be detected, because they extend into the fourth direction of space. What quantum physicists usually call photons are just the 'A-lines' extending into the fourth dimension along the transverse wave fronts' path that correspond to the changing magnetic vector potentials. Photons are always and only virtual until one wave interacts with another or the transverse electromagnetic wave front comes into contact with a material body of just the right size, at which time they become real physical quantities by virtue of the interaction and only by virtue of the interaction. In other words, the interaction creates the photon at exactly that point in space and moment in time. Otherwise photons are not real at any other place in space or time as many quantum theorists propose.

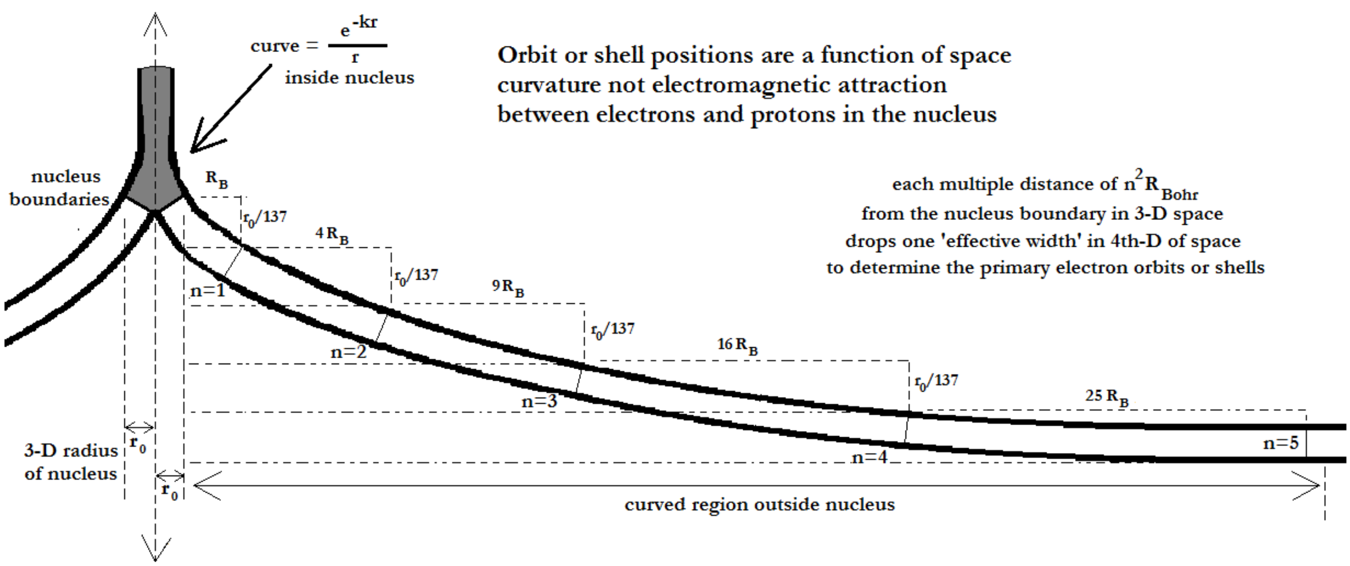
Many scientists have already studied this second term ($mv \times \Gamma$) within the context of both the Newtonian and Einsteinian (Einstein-Cartan geometry) formulations. The Newtonian form was first stated by Oliver Heaviside in 1893, while Einstein developed an alternative unified form of general relativity using the Einstein-Cartan structure of space-time in 1929. Until now Heaviside's Newtonian form of the expression and the later Einstein-Cartan geometry have not been linked together. A few scientists refer to the newly added term as 'torsion', but its correct name should be 'twist' after the concept's real originator W. K. Clifford. (1873, 1878) 'Twist' is a real attribute of each and every dimensionless point in three-dimensional space, whether space is considered internal or external to material particles and bodies. When collectively applied to extended material particles, it is called the property of 'spin'. The 'A-lines' within extended particles that represent all of the geometrical points form 'bundles' that have the same 'twist' to form particle 'spins'.

Other scientists also (wrongly) refer to a separate 'torsion' field, but the term actually applies to a part of the overall gravitational field and should thus be referred to as the 'gravnetic' portion of the curvature. Since twist is a property of every dimensionless point in three-dimensional space, gravity must be affected by twist and twist must be accounted for in gravity theory. Therefore, adding a new term to Newton and Einstein's equations is philosophically justified just as much as observationally necessary to explain Dark Matter and Dark Energy. The terms 'gravnetic' and magnetic thus refer to the individual dimensionless points in three-dimensional space because they are essentially real four-dimensional 'things', while gravity and electricity refer to the metric or extension-geometrical curvature as completely three-dimensional 'things'.

This field structure thus solves two problems in atomic theory. The first is the source of the electroweak force in the SOFT model of the nucleus. While the Yukawa potential field and the strong nuclear force are explained by the four-dimensional metric contact surface between nucleons, the electroweak force reduces to the point geometry of that surface. In other words, the weak nuclear force refers to how Kaluza's A-lines cross or cut through the metric surface between nucleons. The other structural problem solved by this new form of gravity, the gravnetic structure, is the existence and placement of electronic orbits or shells. The gravnetic structure of the atom determines the position of electrons moving at specific speeds around the nucleus and the wavelengths (or frequencies) of light emitted or absorbed by orbiting electrons during transitions just conform to the structural requirements of the geometrical curvature – neither the wavelengths (or frequencies) nor the electrical characteristics of atoms that were used by Niels Bohr to develop his model of electron orbits determine the particular orbital paths of electrons.

The prescribed – allowed or restricted – electron orbits already exist structurally in the space (along the curvature) surrounding the nucleus before the electron jumps into (or out of) another orbital position. In one sense the positions are etched into the curvature in the local space near nuclei by the quantum conditions of existence, but in another sense they are the product of a physical negotiation between the nucleus and the orbiting electrons. Otherwise, they are virtual until an electron falls into any particular orbit. This point would never have been made with regard to previous theories or theoretical models of the orbits, which Bohr only defined after the fact by electromagnetic and quantum considerations alone.

The positions where the orbits occur are gravitational (curvature) equilibrium positions between the nucleus, electrons ($m\boldsymbol{v}$) and the rest of the universe ($\boldsymbol{\Gamma}$), that are modified by the quantum.



Each time the curvature in the fourth direction of space drops by the 'effective width' of three-dimensional space (it experiences a quantum drop of $r_0\alpha$ or about $r_0/137$) such an equilibrium position is established. This structure turns the continuously sloping curvature into energy 'steps' on which the electrons move up and down the curvature as they absorb and emit energy in the form of electromagnetic waves. These equilibrium positions occur at distances of n^2R_B from the center of the nucleus where n=1, 2, .3 and so on. The quantity R_B is the Bohr radius of the hydrogen atom and is equivalent to $h/2\pi m_e c\alpha$ or 5.29×10^{-11} meters. These orbital positions or steps end when the curvature from the nucleus merges with the collective curvature of nearby surrounding matter and/or the universe. Beyond this position the electron would only be subject to the normal gravitational attraction of the nucleus (F=mg). An electron's orbital speed will have gone to zero past this point and its 'free' speed would only be determined relative to other material bodies in the 'free' space between atoms and other material objects.

Given the new equation for gravity, the dependence of the electron orbital or shell structure on gravitational curvature is easy to demonstrate. The normal Newtonian force of gravity (**F** = m**g**) can still be ignored within the atom, but the second term regarding the orbit speed cannot be ignored. By substituting DeBroglie's equation for a matter wave, the gravnetic part of the gravitational force becomes

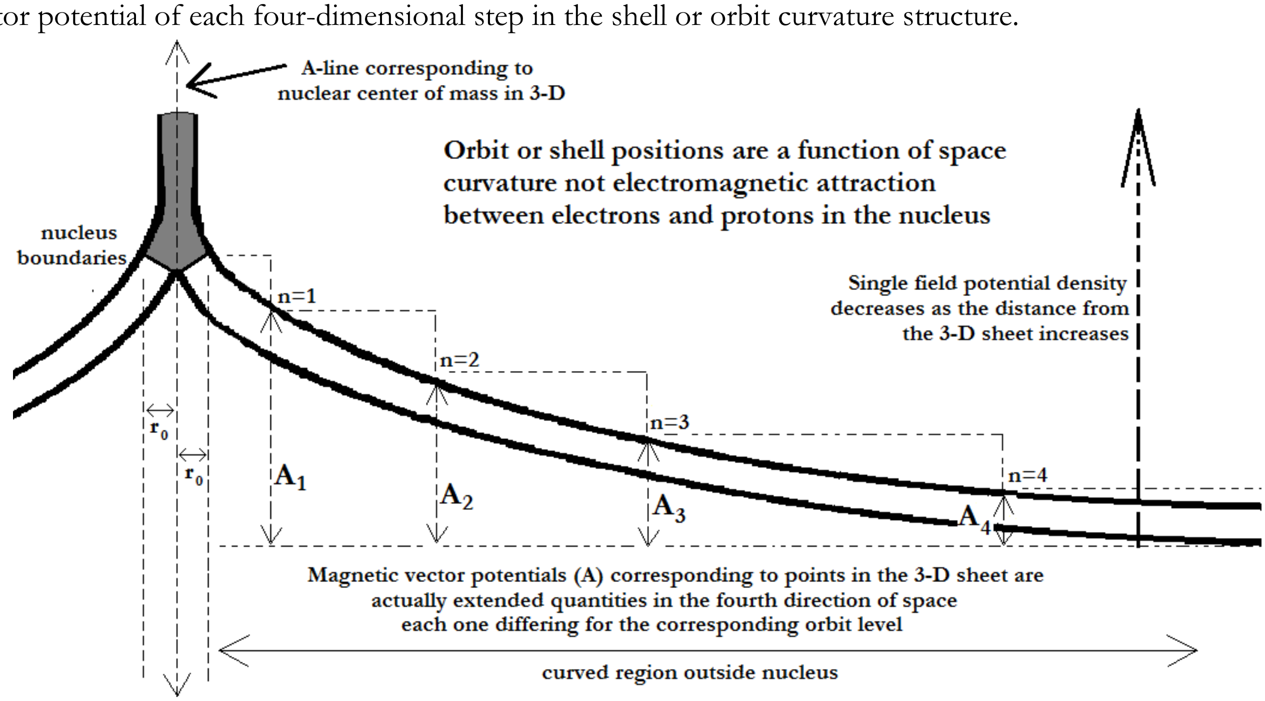
$$\frac{h}{\lambda} \otimes \vec{\boldsymbol{\Gamma}}.$$

This equation readily implies a quantum structure for the orbits such that

$$\frac{h}{n\lambda_v} \otimes \vec{\boldsymbol{\Gamma}},$$

where n= 1, 2, 3, …. Each succeeding orbit is 'n' matter wavelengths ($\boldsymbol{\lambda}_v$) of that orbital electron at the speed the electron travels in that orbit. In other words, the wavelength $\boldsymbol{\lambda}_v$ is not constant from orbit to orbit, but varies from one orbit to another. This equation is essentially quantized by definition since the various levels of electron orbits in the atom represented by the principle quantum numbers (n = 1, 2, 3 …) already correspond to whole wavelengths of the electron matter wave.

The absorption and emission of 'photons' or quanta of electromagnetic waves pose a completely different problem when viewed from the four-dimensional perspective, although the process is still associated with specific wavelengths in a manner similar to the DeBroglie matter waves of electrons in specific structurally chosen quantum levels. The absorption and emission of specific frequencies or wavelengths of electromagnetic waves necessary to allow electrons to jump between orbits does not depend on the length of orbital circumferences (paths) as do the electron matter waves, but rather on magnetic vector potential of each four-dimensional step in the shell or orbit curvature structure.



The potential (energy) difference between each structural step is just that predicted by the Bohr model to account for the emission and absorption spectra of the atoms. These steps restrict the allowable orbits according to the quantum conditions of space at those points in space along the fourth-dimensional curvature surface where the electromagnetic waves or photons come into contact with the orbiting electrons. It may seem that each quantized step in the fourth direction of space is equal because each rises by the same factor of about $r_0/137$, but the density of the single field in the fourth direction of space decreases the further the distance from the primary three-dimensional sheet in which matter is formed by curvature. So the potential differences of each step are different.

As an incoming electromagnetic wave comes into contact with the outermost edge of an atom, it must have enough energy to penetrate the outer shell boundaries of the atom, i.e., climb up the gravitational potential curve. So only electromagnetic waves with high enough energy to be absorbed or pass through the atom actually penetrate the inner confines of the atom and only higher energy electromagnetic waves are absorbed the closer they come to the nucleus. The n=1 energy level is the lowest (rest) energy state relative to the nucleus, but it is also the highest energy level with respect to the overall average curvature (represented by the quantity $\mathbf{\Gamma}$) of the space-time continuum. Therefore it takes greater amounts of energy to liberate electrons from the lower n=1 quantum level (the ground state of the atom) as it does from all lower levels. In all cases, the electromagnetic field and the waves traveling through it are constrained to the gravitational field structure corresponding to the curvature of the space-time continuum.

If the incoming electromagnetic wave has exactly the right amount of energy, neither too much nor too little, the wave just makes it over that particular step along the curvature surface to be absorbed by the electron. However, the wave does not 'know' the energies of the previous steps that it passed over, so only the electron can choose which particular incoming wave to absorb to jump from one orbit to another. The electrons do 'know' or rather determine what energies to absorb to jump between orbits because those orbits are mediated or negotiated by the gravnetic interaction between the nucleus and the electrons according to the condition that the DeBroglie wavelengths of the electrons in any given orbit are quantized in whole number wavelengths according to their speeds. These particle orbits establish fixed positions corresponding to specific and constant values of the electromagnetic vector potentials in the higher dimension of space to which electromagnetic waves must conform during the processes of both emission and absorption. In other words, the electron orbits establish equilibrium positions between electromagnetic and gravitational interactions between the nucleus and electrons relative to the rest of the matter in the universe and light waves are absorbed or emitted according to those restrictions.

Beyond the confines of the atom, the motion of particles and other material objects conform to a wholly different set of conditions dictated by the space-time continuum and the external forces that act on them. Those forces are either gravitational or electromagnetic in nature. The study of motion is quite old and remained unchanged for several centuries after Isaac Newton published his laws of motion. In fact, they remained essentially unchanged until Albert Einstein developed new rules for the electrodynamics of moving material bodies and the concept of special relativity in 1905. While special relativity is very well understood and relativistic limits to motion have been incorporated into the quantum theory, the electrodynamical situation has now changed with the incorporation of a fourth dimension of space, warranting a reevaluation and new analysis of what happens when material bodies move through the five-dimensional space-time continuum.

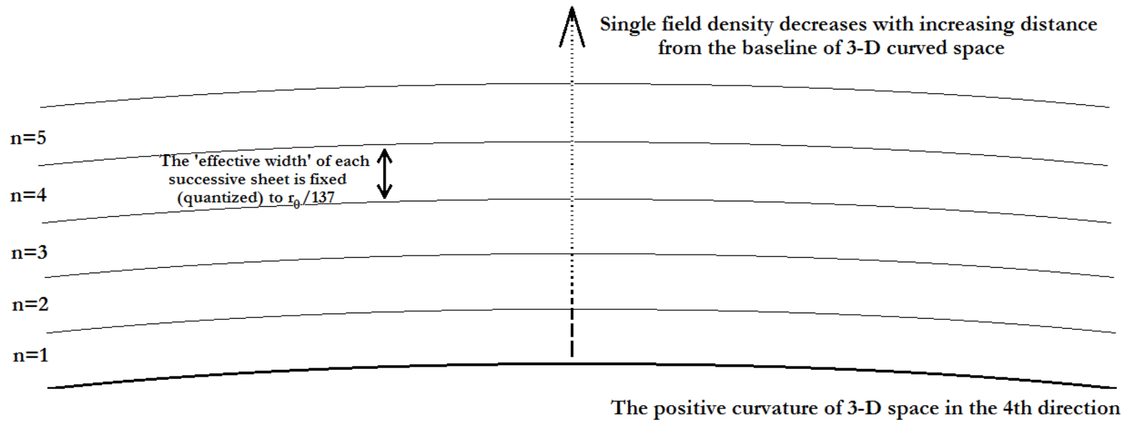
The true dimensions of physical reality

In some respects the single field and five-dimensional space-time are mathematically and perhaps even physically indistinguishable, which begs the question whether or not they are inseparable. They are not, however, inseparable because a primary difference between the two can be specified. It rests in the simple fact that the single field varies in density from one position in four-space and time to another, but the density clumps (particles) and variations are apportioned (by Planck's constant h) and relative (by the speed of light c) to both the quantization and geometry of our normally experienced four-dimensional space-time. Normal space-time is essentially the collection of dimensionless points from which it is constructed. The material extensions in normal space-time that we call elementary particles are thus arbitrary (in the sense that the original creation of elementary particles occurred at random positions throughout the full extension of three-dimensional space) and limited by the quantized geometry of five-dimensional space-time.

However, these extended material bodies are given meaning, relevance and limit by the single field which occupies the whole extent of space-time as characterized by the quantum (h), electric permittivity (ε_o) and magnetic permeability (μ_o). In other words, elementary particles are governed in their most basic interactions as well as their original evolution (creation) by the two physical constants – Planck's constant (h) and the speed of light ($c = (\mu_o \varepsilon_o)^{-1/2}$). Planck's constant is a property of the space-time continuum while the speed of light is a property of the single field that fills the space-time continuum. They combine together to yield our physical and material reality. The permittivity ε_o is the binding constant for the three-dimensions of normal space, the permeability μ_o is the binding constant of normal three-dimensional space to the fourth dimension of space – they are the constants that characterize the single field – and Planck's constant is the binding constant of four-dimensional space to time.

Five-dimensional space-time is restricted by the continuity of the single field while the single field is characterized by a varying density. The three-dimensions (in a sense a 'sheet' or thin film) of normal space that serve as a platform for our commonly sensed material world to play out its existence are actually just the densest portion of the single field. The single field density does not vary over the three dimensions of normal space except according to the restrictions placed on material particles and bodies by Planck's constant and the speed of light, but instead varies linearly in the fourth direction of space. It decreases in density as the distance from the three-dimensional portion of the embedding four-dimensional space increases. However, the 'sheet' is defined and limited mathematically by an 'effective width' of quantum proportions in the fourth direction of space.

The 'effective width' would be approximately $r_o/137$, the radius of a proton (r_o) times the fine structure constant ($\alpha = e^2 c \mu_o / 2h \approx 1/137$). The 'effective width' defines how the quantum limits material and material/electromagnetic interactions and thus our common material and discrete existence. The continuum consists of nearly an infinite number of sheets stacked one upon the other in the fourth direction of space, each representing a different principle quantum number (n=1, 2, …), like the pages in a book.

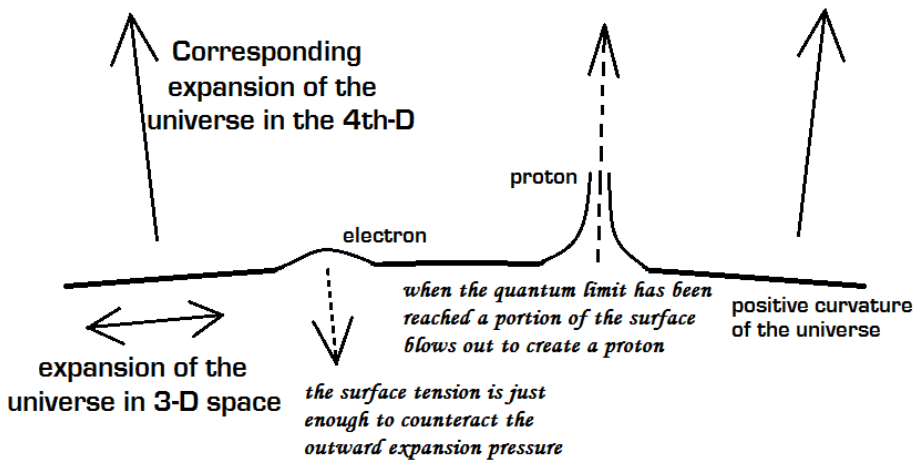


The ‘sheet’ which serves as our common three-dimensional space is represented by principle quantum number one. The ‘sheet’ curves spherically in the macroscopic domain to render the Riemannian curvature of our three-dimensional universe as a whole as proposed by general relativity, while there are three basic forms of curvature in the smallest possible local portions of the ‘sheet’ at the quantum level. They define the primary elementary particles.

The first form of curvature occurs when the ‘sheet’ just begins to curve locally, forming burble (turbulence) that corresponds to a neutrino. A neutrino is the minimum possible localized curvature that fulfils the quantum and geometrical conditions of space-time. The second occurs just before the local curvature becomes great enough to actually fold upon itself, and is called an electron, while the third case occurs when the ‘sheet’ actually folds upon itself locally and forms a proton. The electron is thus the maximum curvature of the ‘sheet’ that fulfils the necessary quantum and geometrical conditions while the proton is altogether different. A proton is created from rupture or tearing (a blowout) in the ‘sheet’ after the maximum of curvature (the electron) is reached due to the differential stress of expansion between the fourth dimension and three-dimensional space is great enough to surpass the maximum curvature allowable by the quantum condition in three-dimensional space. The quantum condition thus manifests in three-dimensional space in the case of the proton as a cap that fixes the diameter of the three-dimensional proton rendering it spherical and limits the extent of the ‘blowout’.

All three of these particles are subject to the same geometrical condition in their extension in the fourth direction of space – the single-polar geometry that imparts them with half-spin. These three geometrical forms represent the true elementary particles as defined by the single field theory or SOFT. Each of these particles can be represented by a ‘field density center’ which lies along Kaluza’s ‘A-line’ corresponding to the geometrical center of the particles in three-space, or rather the extension of a particle's classical center of mass in four-space. While this central ‘A-line’ is important for mathematical reasons and represents the point-particle in quantum theory for calculational purposes, particles are not really points in any of the four dimensions of space but extended portions of curvature in the ‘sheet’. Point particles, as used in the quantum theory, exist merely for mathematical purposes of calculation.

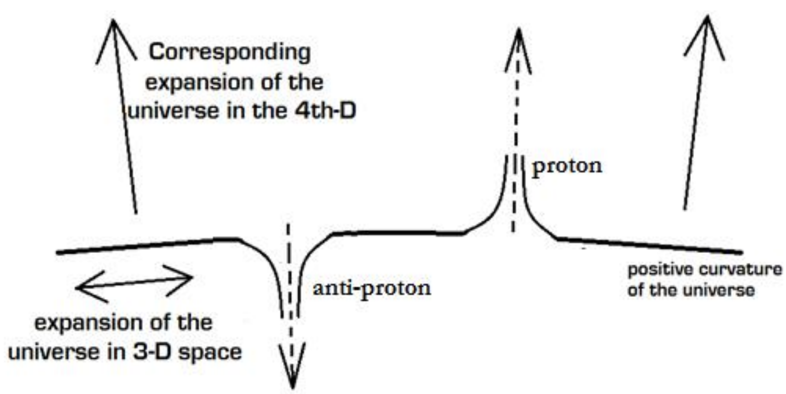
This structure guarantees the existence of a proton-electron dominated universe, explaining why there are very few (if any) anti-particles left over from the Big Bang. If the Big Bang caused the three-dimensional universe to expand internally as well as expand into the higher fourth dimension of space, then the global surface of the universe would act in manner similar to an expanding three-dimensional bubble with a two-dimensional surface.



No anti-particles (oppositely directed bumps and hills) are created while the creation of protons and electrons slows expansion enough to end the inflationary period

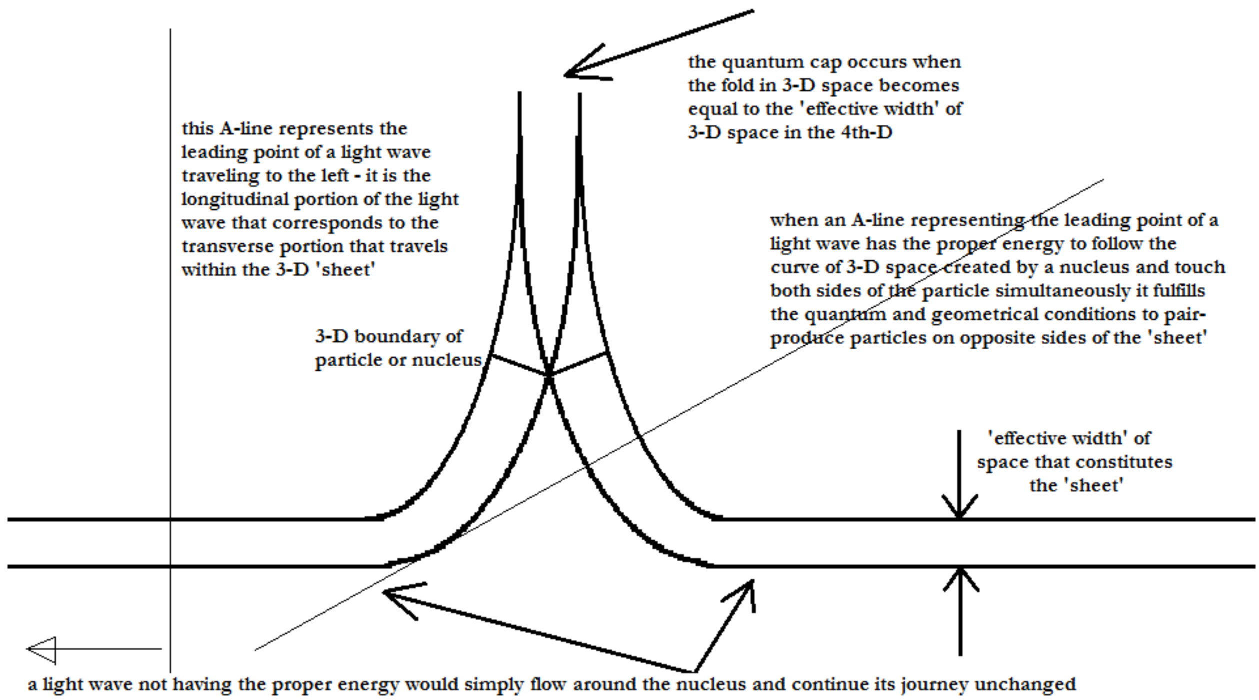
When the single field density of the surface reached a specific minimum value determined by the quantum and speed of light, individual points in space would have erupted outward (blown out not in) along the direction of expansion in four-space creating protons. This would be equivalent to the intersection point when three-dimensional space is folded in each of its dimensions simultaneously. A quantum cap would stabilize the blowout in three-dimensional space and the fourth dimension as well as smooth out the singularity in the curvature predicted by general relativity at the particle's mass center, thus guaranteeing that mathematical singularities (infinities) cannot exist in real physical space-time.

During this initial inflation-ending event, no anti-particles could have been created because anti-particles curve inward toward the center of the expanding bubble and thus against the 'momentum' of the outward direction of the expanding universe. Anti-particles display the same curvature, footprint and characteristics in three-dimensional space except for the single fact that their curvature is away from the expansion of the universe on the opposite side of the three-dimensional 'sheet', which gives them their characteristic opposite electrical charge. Anti-particles are inherently unstable in our universe because of their charge and oppositely directed internal stress, but they cannot decay until they come into contact with their oppositely curved particles.



When an anti-particle and particle make contact and come together, occupying the same position in three-dimensional space, they annihilate or rather cancel out the quantum and geometrical conditions which defined them as particles. The energy that was packaged within their curvature cannot be annihilated, so it is lost as gamma rays that have no curvature. The annihilation works like opposing wave pulses traveling in opposite directions along a single tightly stretched string. The only difference is that the two pulses in the string cancel each other only momentarily and the energy that they carry continues as before and the pulses reemerge after the collision still traveling as they were before they collided. Annihilation is the same as the oppositely directed pulses cancelling out, however, unlike the pulses in the string the particles covert to energy when they make contact and thus disappear.

Pair production creating anti-particle/particle pairs operates in the opposite way and could not have occurred either before or during the particle creation stage that ended the inflationary period of the universe.



Under those special conditions where a gamma ray of the correct energy comes close to a nucleus (or other object corresponding to sharply curved space) and actually surrounds the nucleus (touches all sides of the nucleus simultaneously) the quantum stress in the space-time continuum creates the particle and its anti-particle. Of course the last condition is having the proper energy to create the curvature in the 'sheet', so curvature packages energy in a sense. This process occurs because the 'photon' which is equivalent to an 'A-line' or longitudinal portion of the light wave cannot exist at two different places in three-dimensional space simultaneously, such as both sides of the nucleus simultaneously. So, pair production could not have occurred before the inflationary period ended with the creation of elementary particles and can only occur afterwards when the proper quantum and geometrical conditions are met.

The extremely rapid rate of inflationary expansion in the early universe slowed drastically with the blowout during the creation of particles – like wind going out of a balloon – ending the inflationary period. The creation of protons alone was not enough to completely stop or sufficiently slow the rate of expansion. The energy in the expanding bubble was still too great and a second almost-blowout occurred. This event would not have had enough energy to create more protons because the surface tension (or energy) of the three-dimensional 'sheet' was enough to counteract the blowouts, point-by-point, leaving small bumps in the surface – electrons. Any further quantum fluctuations in the single field that rose to the levels set by the quantum and geometrical conditions would merely have only caused excess 'burble' (turbulence) in the sheet' – neutrinos. The period of cosmic inflation would certainly have slowed sufficiently at this time and no anti-particles would have been created since the motion of the universe expanding outward created the outward protrusions of curvature that are protons, electrons and neutrinos.

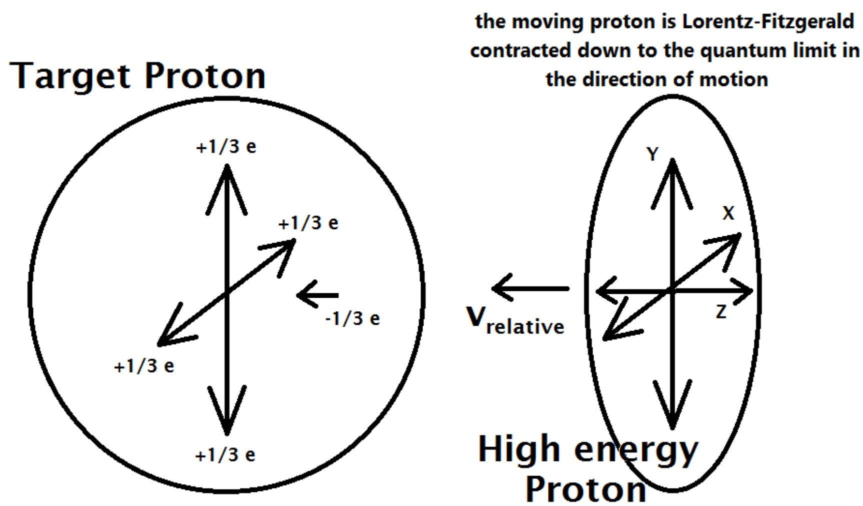
The neutron is not elementary, but rather a compound particle. It forms from the combined curvature of an electron and proton plus a little something (curvature) extra whose 'field density centers' fall along the same central 'A-line'. The central 'A-line' of the proton and electron 'twist' parallel to each other and thus overlap without coupling end-to-end forming a single continuous 'A-line', but also giving the neutron the half-spin necessary for making it a full-fledged particle. Otherwise, that something extra holds them together and has a half-spin in the opposite direction cancelling out one of their half-spins. This little something extra is a neutrino and its central 'A-line' connects to the 'A-line' of either the proton or electron to effectively give the neutron one single continuous 'A-line' with half-spin. So any coupling of 'A-lines' must come from a new unspecified 'A-line' of opposite twist or spin, in order to conserve the parity of the combination. Pseudo-particles form as intermediary energy states when the half-spins of real particles connect to form either a double-polar connection in the higher space (with a spin of one) or cancel each other (forming a spin zero) altogether in situations that correspond to a temporary particle or energy resonance with no 'twist' in the 'sheet'.

Given the process of particle creation, protons are not made of quarks as is claimed in the Standard Model of quantum theory. In fact, quarks do not exist as particles at all, but only as the internal dimensional structure of protons and neutrons as well as some temporary pseudo-particles. Pseudo-particles are just intermediate resonances in the single field that fulfill at least one and as many as several of the conditions for real particle creation, but not quite all of the conditions necessary for particle creation. Therefore they destabilize rapidly and decay into real particles and/or gamma rays. In other words, pseudo-particles are just intermediate energy states or single field density resonances that occur during real particle creation. Quarks are another question altogether. They are only detected during high energy collisions because specific geometric and quantum conditions have been reached for their detection, but they are not real particles.

Quarks are dimensional preferences not particles

During particle collisions of very high energies, a three-sidedness to particles and nuclei has been detected. This three-sidedness has been interpreted as evidence for the existence of particle quarks. But it has nothing to do with creating individual new particles. The three-sidedness reflects the inner dimensional structure of particles and nuclei, or at least how the electrical stress inside the particles distributes itself in reaction to the exchange of energy in the form of compression during the collisions. Take for example a proton-proton high energy collision. If the incoming proton has reached a high enough speed (kinetic energy) to Lorentz-Fitzgerald contract to the 'effective width' of the three-dimensional 'sheet' (a condition for quantization of a spatial dimension) before colliding head-on with the target proton, the internal pressure on the target proton will be great enough to cancel the internal proton's outward electrical stress along that direction of space.

According to Pascal's Principle, if the internal electric field stress were to act like a continuous fluid it would normally be split up equally along the three-dimensions of space that the proton occupies. But the inward counteracting pressure of the collision causes the internal electrical stress to split equally in the two dimensions perpendicular to the direction of the incoming colliding proton.



The collision thus results in an inward pressure of -1/3 along the direction of motion and outward stresses of +2/3 and +2/3 of the electrical charge along the remaining two dimensions or directions of space inside the target proton. This configuration thus accounts for the −1/3, +2/3 and +2/3 electrical charge of 'quarks' from which the proton is constructed according to the Standard Model. This dimensional model of 'quarks' also explains why 'quarks' can never be found outside of the particles and nuclei in which they reside, solving one of the major dilemmas associated with 'quark' theory.

In this more realistic 'quark' model, positive fractions or ratios of the fundamental electrical charge indicate outward electrical stress while negative ratios indicate inward electrical stress or electrical stress compression. Total electrical stress (charge) must equal one unit (e) within particles and the resulting strain in space surrounding charged particles. The external strain in surrounding space due to the internal electric stress is commonly called the electric field. The fact that the internal electrical stress breaks down into ratios of n/3 is simply due to the fact that material space is three-dimensional, which should render this particular explanation all the more obvious. In other words, the quantum of electrical charge 'e' is just a strain in the surrounding space caused by the internal electrical stress of a real extended particles and a quark is just how that stress is *redistributed along the three dimensions of space* within the particle – according to a quantized Pascal's Principle – during a high energy collision or interaction with another real extended particle. So quarks are not particles in themselves, but rather the differently apportioned internal dimensions of space.

If this new model of particles is accurate, then the quark-gluon model of hadrons and atomic nuclei is in serious jeopardy and most physicists' commonly accepted central ideas regarding the fundamental nature of matter falls apart. However, this presents no problem at all for the five-dimensional space-time model. If protons and neutrons can be modeled as (and really are) minute bumps and hills (three-dimensional curves) of curvature in four-dimensional space, then a nucleus would consist of the three-dimensional particles stacked one on top of the other, like paper cups, in the fourth spatial direction. It is commonly known that two bits of matter cannot simultaneously occupy the same position in three-dimensional space, but there is no equivalent prohibition regarding their being stacked in a fourth direction of space, a fact which this model uses quite freely to explain nuclei and compound particles such as the neutron.

So, from the four-dimensional perspective of the neutron, the electron and proton are stacked one upon the other in the fourth spatial direction and thus occupy the same space in the normally experienced space-time continuum. The proton and electron curvatures are additive producing the curvature associated with a single neutron, but the proton and electron cannot properly come together to form a neutron until the electron assumes the same four-dimensional configuration as the proton – it must assume a blown-out configuration in the fourth direction of space. Since the electron already has the maximum curvature possible before blowing out it only needs to add a minimum of curvature to blow out and join with the proton to create a neutron. That minimum corresponds to the curvature of a neutrino, so a neutron is really a proton, electron and neutrino that are stabilized and lose their individual identities within the nucleus when their individual curvatures add together.

Outside of the nucleus the combination is unstable (unnatural), so the neutron decays into its three fundamental real particle components. The instability of the free neutron is due to the overlapping or coincidence of the two independent 'A-lines' – or rather the four-dimensional space-time curvature at the quantum level tends toward or favors the three natural extremes of local curvature that define the neutrino, electron and proton. Otherwise the 'stacking' structure of the electron and proton in the fifth dimension to form the neutron implies the basic structure of the atomic nucleus.

Of course, quantum theorists and philosophers do not discuss this possibility because it makes no difference to the mathematics and the mathematics is, after all, what quantum theory is all about. Ordinary logic be dammed, unless, of course, it's mathematical logic. Of course, at some point the concept of mathematical point-particles must 'converge' to the concept of an extended-particle universe. The pseudo-extension of particles that results from fields of probability-density completely fails to create material extension, even though the mathematics works in solving problems and making predictions. The simple mathematics of the Heisenberg Uncertainty Principle that is normally invoked, on the other hand, does not even require the existence of extended particles. The word 'particles' is just a 'handle' that quantum scientists find necessary to bridge the tremendous gap between the mathematical concepts and commonly experienced material reality. In other words, the word 'particle' is normally used by quantum theorists to describe something that they do not really understand, *i.e.*, the difference between real material extended particles whose existence has never been questioned because their existence has been verified by common observation and the 'material' entities which they alone envision through mathematically blinded eyes.

Quite frankly, modern physicists, scientists and philosophers are 'barking up the wrong tree' with regard to quarks. They see the basic problem of reality as a dichotomy between discrete quantum reality and a continuous relativistic reality, so quarks must be particles (and nothing else) in spite of experimental evidence to the contrary. In other words, they are completely missing the real problem of point and extension in either space or time. In so doing, what quantum theorists deem the inherent uncertainty in nature is no more than nature informing scientists through experimental measurements that the mentally produced logical equations by which they describe nature and model reality, as it might be, do not match the intuitive physical models that they have likewise created. Just because a physical quantity or mathematical term appears in an equation, there is no guarantee that the quantity represents a real physical and/or material particle. It only demonstrates that the equation does not match the 'experimental' reality that the scientists themselves have created. No clearer evidence of this can be found in anything other than the need to redefine the concept of 'particles' to fit the predetermined and mistaken vision of what quarks should be, rather than what they really are.

VII. History trends in this direction

With Einstein dead (1955) and no one else alive to carry on and complete his vision of his unified field theory, unification based on general relativity all but died away while the quantum theory and relativity continued to follow their own evolutionary courses under the assumption that they are completely and irrevocably incompatible, which is another of the great 'phallacies of fysics'. During the 1960s and 1970s, a number of advances (QED, QFT, QCD and etc) convinced quantum theorists that unification was a good thing, but this new unification was based on the quantum and sought to overthrow or replace general relativity with a whole new physical model. However, the quantum theory will never overthrow relativity and relativity will never replace the quantum because they are fundamentally different "things" or approaches to understanding nature and material reality – they form a necessary duality in nature.

Relativity is first and foremost about form (structure) and the quantum is primarily all about function, which come together as one of the most fundamental dualities (known as non-commuting quantities in physics) in nature, but there is always a bit of each in the other. However these two ideas, form and function, are not necessarily incompatible since there is always a little of one in the other at a higher level of understanding.

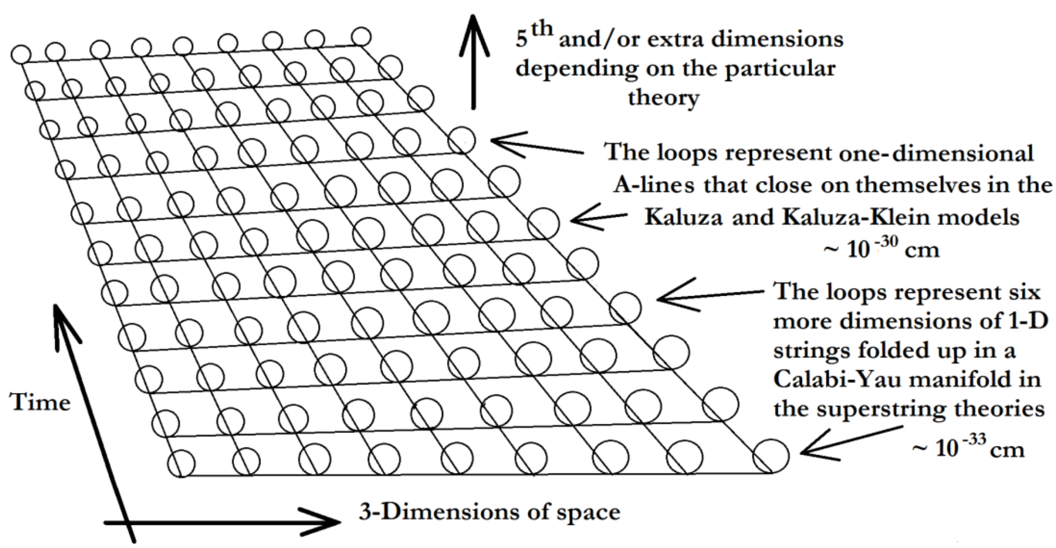
However, these quantum attempts at unification (the so-called TOEs) are also incomplete and fall prey to the same problem of distinguishing between point and extension that ultimately doomed the attempts to base a unified field thoery on general relativity

Quantum theorists claim particles are points (discrete) in fields (QFT) rather than extended bodies because they cannot deal with the concept of continuity – yet their fields are continuous. In reality, it is not the concept of continuity that quantum theorists cannot deal with, it is the concept of extension because extension implies a geometry of space and the quantum theory is a spatially non-geometric theory. Quantum theory deals with points in some form of unspecified space rather than a specified space which is defined by the points that constitute that space. Points are nothing, literally, so they cannot be particles, while particle theorists still cannot explain how to go from their point-particles to observed extended particles without inventing unnecessary new particles, ad infinitum.

In fact, the whole concept of 'discrete' particles is physically impossible. For a point or a particle to be 'discrete', it would be a 'no-thing', something beyond human knowledge and reckoning that cannot even be given a name because naming it would give is a property that if could not possibly have. A 'no-thing' would literally have no connection to either the space in which we live and experience or any experience by which science could even know of its existence. Unfortunately, quantum theorists misuse the term 'discrete' to mean a nothing with no physical properties, which is different than a 'non-thing' that is impossible. They use the term discrete to mean a nothing, a zero point on a line that has no other characteristics beyond its value of nothing, and that includes both the properties of extension in space and/or mass. By so misusing and even abusing the term 'discrete', quantum theorists have falsely structured and interpreted quantum theory in such a way that it has been falsely rendered mutually incompatible with relativity theory, thus making it impossible to unify the two even though nature unifies the two on a constant basis.

Even in the most advanced quantum-based theories, the differences between true 'discrete' and a 'discrete' nothing have been so mixed up that the ensuing mathematical beauty of the theory has become its main and possibly only claim to fame. The superstring theory emerged in the 1980s and 1990s as possibly the best chance to unify gravity, the quantum and the rest of physics. But the superstring theories have run into innumerable difficulties, the least of which is that their theory is completely non-falsifiable and unable to make trest4able predictions. The so-called mathematical beauty of the theory notwithstanding, these theories have repeated the same mistake of not finding a method to connect the points in three-dimensional space or four-dimensional space-time with other such points to constitute a measurable extended space of object.

Each point in normal space or space-time is actually a six-dimensional Calabi-Yau manifold of vibrating one-dimensional strings. The vibrations of the springs and their relative resonances generate physical properties in the discrete points of space or space-time what give to the normal physical 'laws' that the universe follows.

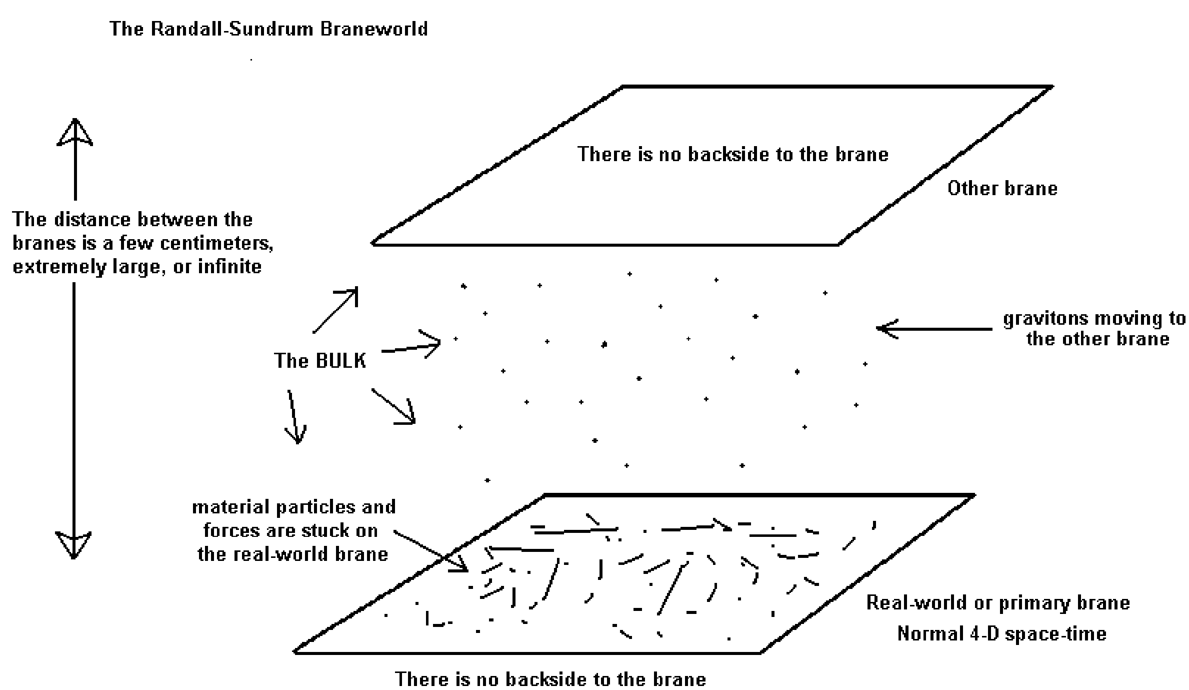


Each intersection represents a dimensionless point in 3-D space and there is no space between points

Yet how can this be so? Quite beyond the obvious question of what is a one-dimensional string and how could it vibrate (?) within the one dimension that it supposedly occupies, the Calabi-Yau bundles fill 'discrete' points in space for which no simple connection or connecting property has ever been defined. That is why there are literally millions upon millions of possible different superstring theories. You can put an infinite number of nothings into a nothing and still get nothing, at least nothing that has any real meaningful connection to our real material universe.

This problem was somewhat alleviated by Edward Witten's discovery of brane theories in the 1990s. Branes are merely two and higher-dimensional surfaces or membranes that are wrapped around to form tubes that extend in another one-dimension as strings. One brane theory in particular has gained quite a large following of adherents if not a lot of publicity and has proved quite popular outside of the scientific community: It is called the Randall-Sundrum Braneworld after its developers, Lisa Randall and Raman Sundrum. This model was developed to demonstrate why gravity is so weak relative to the other fundamental forces (interactions), yet so important as to bind all of the stars and galaxies in the universe together. This is called the hierarchy problem or paradox. To solve this paradox, the Braneworld model posits two four-dimensional branes that are parallel to each other and suspended within a five-dimensional bulk.

The bulk seems to have no other purpose than the convenience it affords the model by accounting for the distance between the two branes. The bulk seems to be no more than a mathematical device that was only mandated to justify the distance between the two branes. The branes themselves have no other features than the fact that they are platforms upon which to place material reality, at least in the case of the primary or real-world brane. Unlike a normal membrane in three-dimensional space, the branes seem to be one-sided and the single side of each brane is facing the other at a constant distance. In one version of the Randall-Sundrum (RS) Braneworld, dubbed RS-1, the branes are a few centimeters apart, while they are extremely distant, but not quite infinitely far apart, in RS-2. In still another version, the RS-1 branes are a very large distance from one another and infinitely far apart in RS-2.



The distance does not really seem to matter, big or small, since its only purpose is to weaken the effect of the gravitons traveling between the two branes.

In either case, our normally experienced material world all of the physics that science uses to describe reality is restricted to only one brane. All points in this primary brane are actually bundled Calabi-Yau higher-dimensional strings, which allow the Randall-Sundrum model to duplicate (or so it is assumed) the standard model of the quantum theory. All of the natural forces (interactions), including electromagnetism, the electroweak and strong nuclear forces (interactions), are restricted to act within or across the primary brane according to the standard model of quantum theory, except for the gravitons (gravity) that travel between the primary or real-world brane and the other brane. The large (non-Planckian) distance between the two branes accounts for the weakness of gravitational attraction relative to the other natural forces (interactions).

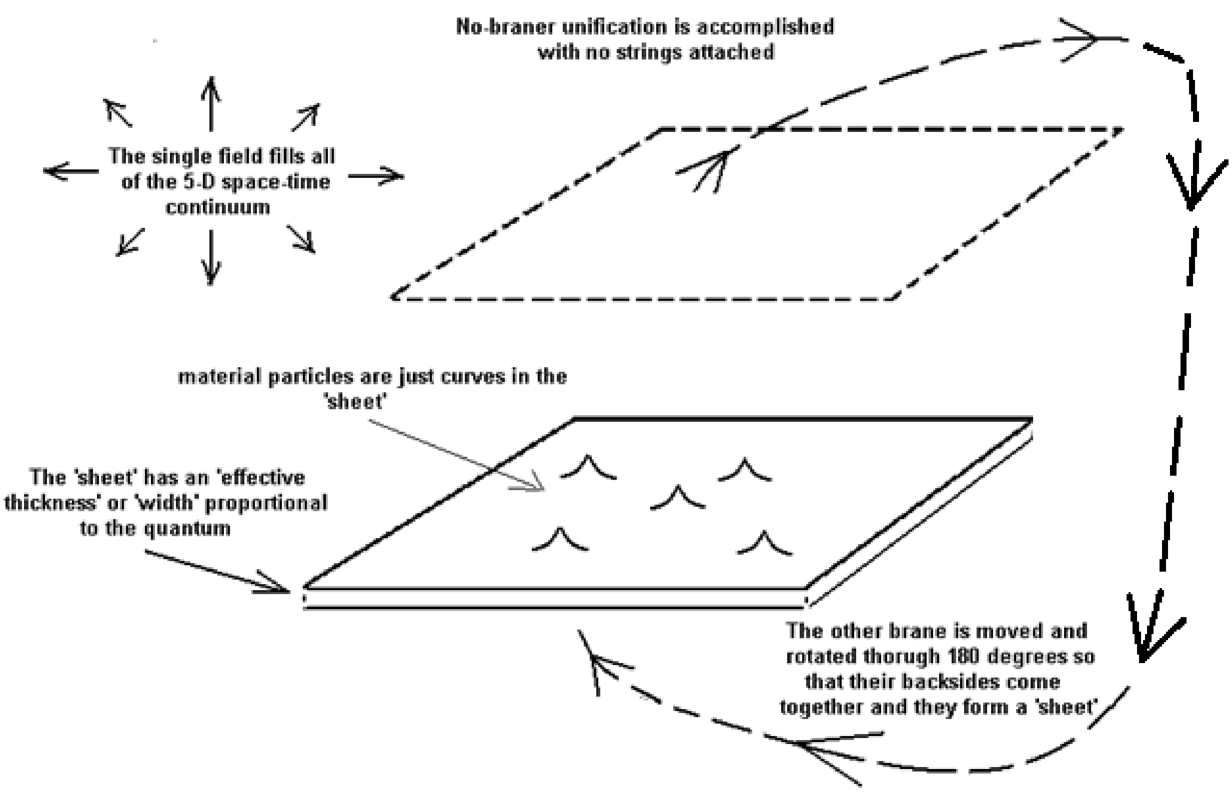
In spite of its beauty (such a beauty as only a mother could love), simplicity (only with a long stretch in the meaning of the term) and popularity (and it certainly is far more popular than it needs to be), this model is irrelevant to physics and the natural world of our experience. The various forms of the Randall-Sundrum model are 'scientifically' untenable even if they eventually turn out to be mathematically viable. But they are not even mathematically viable at present and mathematical beauty is an illogical personal opinion that is irrelevant in science. It is bad enough that all of the problems stated above for superstrings in general carry over into all of the brane models, but this particular model breaks nearly every rule in the book of nature, at least so far as acceptable science is concerned. The bulk is totally undefined except for its dimensionality. The branes are also undefined and are not even analogous to real material membranes because they are one-sided and have no thickness in the fifth dimension that characterizes the bulk. Nothing is said about the extent or shape of the branes in the space-time that they occupy.

It would seem that they must be parallel and thus Euclidean flat, which would imply that they are infinitely extended in the normal four-dimensions of space-time. Nothing is said about what exists on the backside of the branes, even though it seems that they should have a backside. The branes are not continuous with each other or the bulk in general, in that the two branes and bulk are three distinct individual and separate 'things'. Under these conditions, this model breaks and completely obliterates Kaluza's first mathematical condition that our four-dimensional space-time be closed with respect to the higher dimension as well as his assumed condition of continuity. For some reason, picturing the Randall-Sundrum Braneworld models evokes visions of the old song "Me and My Shadow", but little else in the realm of physics.

The superstring and thus brane theories are supposed to be dependent upon Kaluza's theory and thus general relativity; at least theoreticians assume so. But there is no space-time curvature associated with either of the Randall-Sundrum branes. In fact, gravitons shoot between the branes, but there are no gravitons in general relativity. At present, gravitons are no more than wishful thinking by those quantum theorists who hope to someday develop a theory of quantum gravity. At least no gravitons have ever been detected despite four or more decades of attempts to verify their existence. It also seems strange that gravitons are usually considered the exchange particles that travel between gravitationally attracted bodies in quantum gravity theories. However, in the case of the Randall-Sundrum Braneworld, gravitons do not travel between material bodies, which are stuck to the primary brane, but between the two branes. So how can gravitons account for the mutual attractions of material bodies in the primary brane if they are traveling between the different branes? So it would seem that the ever so popular Randall-Sundrum Braneworld model is little more than science fiction, and not even good science fiction at that. In spite of what some scientists have claimed, two branes are not better than one.

In fact, the basic Randall-Sundrum model is not even original. The American astronomer, Simon Newcomb, developed nearly the same model in the late nineteenth century. For Newcomb, the central problem was explaining the null results of the Michelson-Morley experiments and other attempts to detect the aether that was thought necessary to carry electromagnetic waves through empty space. He envisioned a truly physical model based on two parallel three-dimensional sheets, curved in a higher fourth dimension. These sheets were a short distance apart within the higher fourth spatial dimension and separated by the luminiferous aether of early electromagnetic theory. His model did not work either so it was never highly advertised, as is today's Randall-Sundrum Braneworld, and is all but forgotten today.

Yet some good can come from the Randall-Sundrum model. In fact it is a step in the correct direction, even if it was taken as a blind step in some non-specified direction. Kaluza's original 'cylindrical condition', or at least a modified version of it, can be restored if the secondary brane is flipped over and spun around such that it's bottom comes into contact with the bottom of the primary brane and they become one single brane or 'sheet' with an 'effective width' within the five-dimensional bulk.



The bulk can then be closed and defined as a single-polar Riemannian embedding manifold. If curvature is then returned to the primary 'sheet', the Randall-Sundrum model corresponds quite nicely with the single field model which has been developed within this paper. This correlation is important because it demonstrates that other scientists, even when they are working from false premises, are slowly approaching the same model as here proposed.

That simple historical fact is important because the quantum theory as a whole suffers from the same point/extension duality as the classical UFTs, while the concept of a point-particle is itself a serious 'phallacy'. So QED, QFT, QCD and the Standard Model are nothing more than useful, sophisticated, extremely accurate but very complex physical 'approximation of reality' methods that do not really represent physical reality as a true theory should. When this is understood, the mutual incompatibility of the quantum and relativity disappears and their common physical characteristics and properties with regard to the dualism of space leads to the same unified model. So in the end the only answer to unification is to merge or blend the various theories together as they are without making major changes (except interpretative) in them. In other words save the good and get rid of the bad in each theory. Science must render the seemingly incompatible quantum and relativity theories complete and compatible by developing a proper geometry that includes both the point and metric as equal partners in our geometric reality.

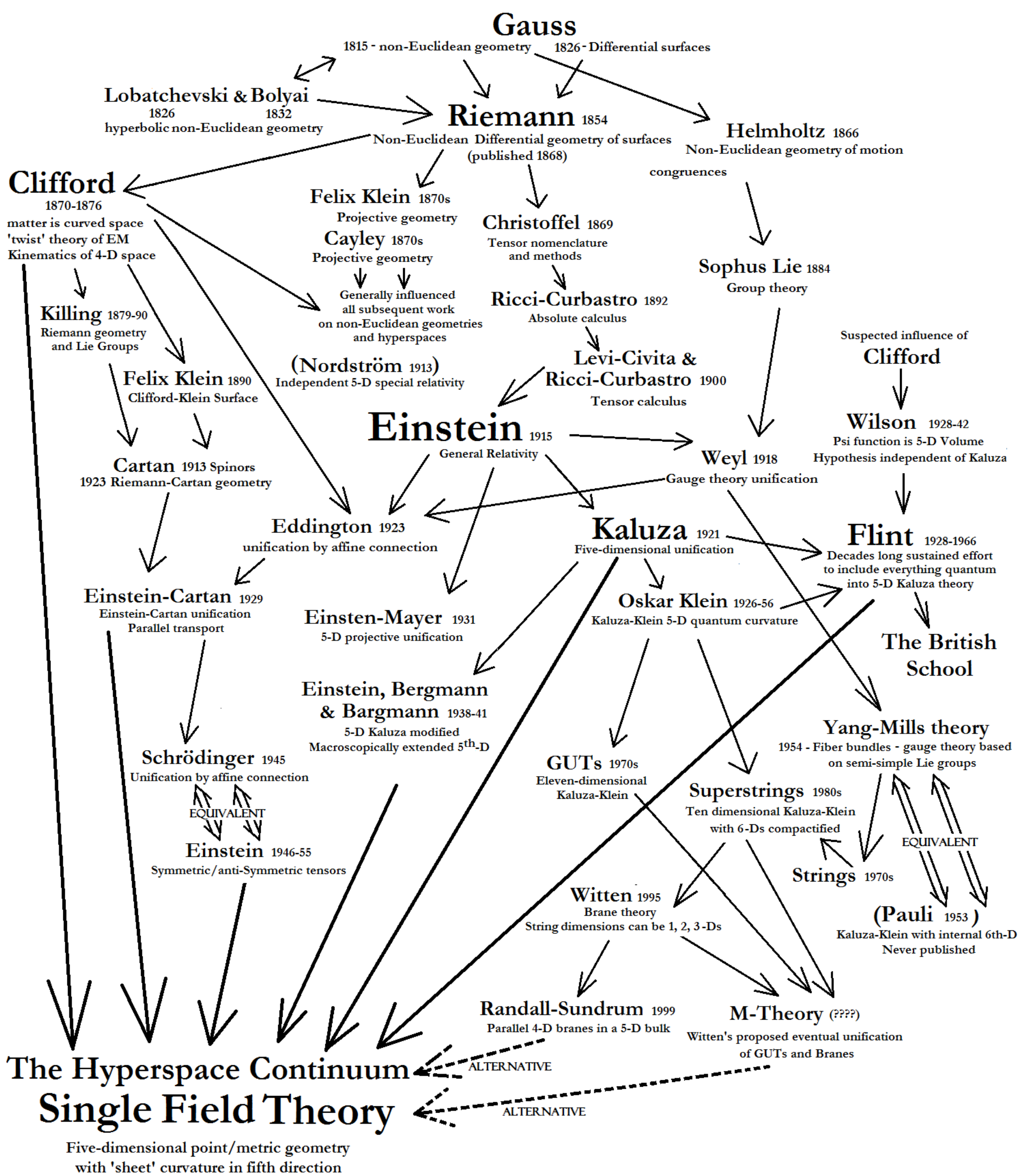
This is necessary since all present theories of physical reality suffer from the same problem by failing to adequately account for how an infinite number of infinitesimal points can yield a continuous extended space-time – even theoretical mathematics is presently facing this same problem. The physical problem is directly related to the mathematical problem of infinitesimals and continuity which has as long a history as the physical problem of points and extent in space and time and in fact the problem goes all the way back to Euclid's original work in geometry two and a half millennia ago.

With regard to the mathematical concepts, in 1949 Weyl (p.44) compared the notions to Galileo's "bending theory" and stated that "*If a curve consists of infinitely many straight 'line elements', then a tangent can simply be conceived as indicating the direction of the individual line segment; it joins two 'consecutive' points on the curve.*" The true scientific reality is that there are two and only two instances that demonstrate the physical reality of infinitesimal dimensionless points – centers of rotation and mass – while these two examples imply the necessity of a dual point- and extension-geometry to explain material reality. Einstein and Bergmann nearly solved the geometrical problem of points in 1938, although they were not attempting to, but stopped short of the answer because they only wanted to get rid of Kaluza's cylindrical condition to render the fifth dimension more realistic.

Einstein and Bergmann developed a geometric proof allowing for a large fifth-dimension of space, but gave no clues regarding the actual geometry of that 5^{th}-D embedding dimension of the space-time continuum. Unfortunately, they also threw out Klein's extension of Kaluza's model to include the quantum when they threw out the cylindrical – Klein's extension was incomplete since Kaluza's model was incomplete and thus superstring theory and brane theory are also incomplete. Yet Klein had the right idea – quantize the extra dimension of space which automatically quantizes four-dimensional space-time. This was demonstrated indirectly by Einstein and Bergmann's 1938 mathematical proof.

The main question then became how to get from the mathematical concept of dimensionless points, an infinite number of which make up any extended line, to a real continuous extended line when two or more points put together by contact still reduce to a single point because they are dimensionless. As this last question was recently solved (Beichler, Vigier VIII 2012 and NPA 2013), the successful completion of Einstein's program to unify all if physics has now been completed. It would seem as if all of physics has been moving in this same direction once the problem of the point has been identified and resolved. This then is the new challenge for physics and all of science. This model can either be taken seriously and tested as is the standard practice in science, or it can be discarded as a new form of scientific heresy. However, this new single field model of physical reality cannot be denied and ignored. The unified field theory is now a done deal. If it is not taken seriously now, it will be in the future. Quite simply, history has demonstrated that there is a simple general principle for unification that cannot be denied because all of physics is headed toward the same idea: Since space and time have two compatible parts – point and extension – which are inherently different but which depend one on the other, all of physics which occurs in our commonly experienced three-dimensional space and time, classical or Newtonian or Einsteinian gravity must also be affected by this dichotomy as are electromagnetism and the quantum.

Unification Tree



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